

Pomerons and BCFW recursion relations for strings on D-branes

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ABSTRACT: We derive pomeron vertex operators for bosonic strings and superstrings in the presence of D-branes. We demonstrate how they can be used in order to compute the Regge behavior of string amplitudes on D-branes and the amplitude of ultrarelativistic D-brane scattering. After a lightning review of the BCFW method, we proceed in a classification of the various BCFW shifts possible in a field/string theory in the presence of defects/D-branes. The BCFW shifts present several novel features, such as the possibility of performing single particle momentum shifts, due to the breaking of momentum conservation in the directions normal to the defect. Using the pomeron vertices we show that superstring amplitudes on the disc involving both open and closed strings should obey BCFW recursion relations. As a particular example, we analyze explicitly the case of $1 \rightarrow 1$ scattering of level one closed string states off a D-brane. Finally, we investigate whether the eikonal Regge regime conjecture holds in the presence of D-branes.

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1. Introduction and summary of results

Recently, there has been remarkable progress in exploring the properties of the S-matrix for tree level scattering amplitudes in gauge and gravity theories. Motivated by Witten’s twistor formulation of $\mathcal{N} = 4$ Super–Yang–Mills (SYM) [1], several new methods have emerged which allow one to compute tree level amplitudes. The Cachazo–Svrcek–Witten (CSW) method [2] has demonstrated how one can use the maximum helicity violating (MHV) amplitudes of [3] as field theory vertices to construct arbitrary gluonic amplitudes.

Analyticity of gauge theory tree level amplitudes has lead to the Britto–Cachazo–Feng–Witten (BCFW) recursion relations [4, 5]. Specifically, analytic continuation of external momenta in a scattering amplitude allows one, under certain assumptions, to determine the amplitude through its residues on the complex plane. Locality and unitarity require that the residue at the poles is a product of lower-point amplitudes. Actually the CSW construction turns out to be a particular application of the BCFW method [6].

The power of these new methods extends beyond computing tree level amplitudes. The original recursion relations for gluons [4] were inspired by the infrared (IR) singular behavior of $\mathcal{N} = 4$ SYM. Tree amplitudes for the emission of a soft gluon from a given n-particle process are IR divergent and this divergence is cancelled by IR divergences from soft gluons in the 1-loop correction. For maximally supersymmetric theories these IR divergences suffice to determine fully the form of the 1-loop amplitude. Therefore, there is a direct link between tree level and loop amplitudes. Recently there has been intense investigation towards a conjecture [7, 8] which relates IR divergences of multiloop amplitudes with those of lower loops, allowing therefore the analysis of the full perturbative expansion of these gauge theories¹. These new methods have revealed a deep structure hidden in maximally supersymmetric gauge theories [11] and possibly in more general gauge theories and gravity.

The recursion relations of [5] are in the heart of many of the aforementioned developments. Nevertheless it crucially relies on the asymptotic behavior of amplitudes under complex deformation of some external momenta. When the complex parameter, which parametrizes the deformation, is taken to infinity an amplitude should fall sufficiently fast so that there is no pole at infinity². Although naive power counting of individual Feynman diagrams seems to lead to badly divergent amplitudes for large complex momenta, it is intricate cancellations among them which result in a much softer behavior than expected. Gauge invariance and supersymmetry in some cases

¹Very recently there has been a proposal for the amplitude integrand at any loop order in $\mathcal{N} = 4$ SYM [9] and a similar discussion on the behavior of loop amplitudes under BCFW deformations [10].

²There has been though some recent progress [12, 13] in generalizing the BCFW relations for theories with boundary contributions.

lies into the heart of these cancellations.

A very powerful criterion has been developed in [14] and studied further in [15], which allows one to infer, purely from the symmetries of the tree level Lagrangian, which theories allow BCFW relations and for which helicity configurations. Considering the deformed particles as “hard” which propagate in the “soft” background created by the undeformed particles, we can study the asymptotic behavior of scattering amplitudes under large complex deformations. This behavior is ultimately related to the ultraviolet (UV) properties of the theory under consideration. For example, $\mathcal{N} = 4$ SYM is a theory with excellent UV behavior and the BCFW relations take very simple form allowing one to determine fully all tree level amplitudes of the theory for arbitrary helicity configurations [16].

It is natural to wonder whether these field theoretic methods can be applied and shed some light into the structure of string theory amplitudes. This is motivated, in particular, by the fact that the theory that plays a central role in the developments we described above, that is $\mathcal{N} = 4$ SYM, appears as the low-energy limit of string theory in the presence of D3-branes. In order to even consider applying the aforementioned methods to string scattering amplitudes, one needs as a first step to study their behavior for large complex momenta. Since generally string amplitudes are known to have excellent large momentum behavior, one expects that recursion relations should be applicable here as well. Nevertheless, one should keep in mind that although the asymptotic amplitude behavior might be better than any local field theory, the actual recursion relations will be quite more involved. The reason is that they will require knowledge of an infinite set of on-shell string amplitudes, at least the three point functions, between arbitrary Regge trajectory states of string theory.

The study of the asymptotic behavior of string amplitudes under complex momentum deformations was initiated in [17] and elaborated further in [18, 19]. These works established, using direct study of the amplitudes in parallel with pomeron techniques, that both open and closed bosonic and supersymmetric string theories have good behavior asymptotically, therefore allowing one to use the BCFW method³. Moreover, it was observed in [18] that for the supersymmetric theories the leading and subleading asymptotic behavior of open and closed string amplitudes is the same as the asymptotic behavior of their field theory limits, i.e. gauge and gravity theories respectively. This led to the eikonal Regge (ER) regime conjecture [18] which states that string theory amplitudes, in a region where some of the kinematic variables are much greater than the string scale and the rest much smaller, are reproduced by their corresponding field theory limits. For bosonic theories there is some discrepancy in some subleading terms [19] which most probably can be attributed to the fact that an effective field theory for bosonic strings is plagued by ambiguities due to the presence of tachyonic modes.

³Recently, pomerons have also been used in [20] to advocate that BCFW relations exist in higher spin theories constructed as the tensionless limit of string theories.

The purpose of the current work is to study string amplitudes which involve both open and closed strings in the presence of D-branes. We will use the method of pomeron operators since we believe it is the most direct one and it will allow us, as a byproduct, to derive a few interesting results which apply beyond BCFW. In addition, the analysis of BCFW deformations is general and applies to all situations where a defect (brane) in spacetime interacts with bulk modes.

In section 2 we give a short review of the relevant CFT machinery for string operators in the presence of a boundary on the world-sheet. In section 3 we use the operator product expansion (OPE) technology at hand to derive the pomeron operators, along the lines of [21], for D-branes in a flat background. The main idea is to divide the string operators into two sets highly boosted relatively to each other. The pomeron operator is exchanged between the two sets of operators and exhibits the typical Regge behavior expected for such a process. We study the Regge behavior of the prototype amplitude for the scattering of two closed string tachyons off a D-brane.

There are two qualitatively different factorization channels of the string amplitude on the world-sheet which lead to two different pomeron operators. The first factorization, into a sphere and a disc, leads to the closed string pomeron operator while the second, into two discs, leads to the open pomeron operator. This factorization can appear even if one of the two sets of operators has a single closed string operator. This is unlike the usual purely closed string theory analysis which requires at least two operators for each set. This is a novel feature due to the presence of the D-brane.

Subsequently, section 2 contains two applications. First, we compute a mixed open-closed string amplitude on the disc applying the pomeron machinery. This example demonstrates an important point. The field theory limit factorization of string amplitudes cannot always be reproduced by a corresponding Regge channel factorization. This is unlike the pure open string theory where Regge channels due to two highly-boosted gluons coincide with the two particle factorization channels of gauge theory amplitudes. Actually for some amplitudes there is no Regge type behavior at all for the world-sheet factorization which leads to the massless poles. This will have profound consequences for the ER regime conjecture in the presence of D-branes.

The second application deals with ultrarelativistic scattering of D-branes. Unlike the previous examples, where pomerons were used for computing scattering of highly boosted string states off a D-brane, in this case we use the pomeron technique to compute the scattering of one D-brane off another. This way we can reproduce the result of [22]. We conclude this section by giving the explicit formulas for the pomeron operators for level one states of both bosonic and superstring theories. There are three different pomerons for each case. The open string pomeron from the OPE of two open string states on the disc, the closed string pomeron from the OPE of two

closed strings on the sphere and a tadpole pomeron operator from the OPE of a closed string with its image as it approaches the boundary of the disc world-sheet.

The results of this section can be useful for holographic computations as well. One would need to extend the pomeron operators in a curved, asymptotically AdS background along the lines of [21]. Open strings attached to D-branes model quarkonium states and closed strings glueballs.. Therefore pomeron operators for open-closed string amplitudes would be useful in studying quarkonium-glueball scattering in the Regge regime. Furthermore, D-branes correspond holographically to non-perturbative states of the boundary gauge theory. For D-brane scattering we would need the corresponding boundary states for highly boosted D-branes moving in the curved background. We leave these interesting applications for future investigations.

In section 4 we give a lightning review of the BCFW method for field theories. We continue with an analysis of the possible BCFW shifts for scattering amplitudes in the presence of defects. The analysis is general and does not rely on string theory. It can be used for field theories in the presence of defects such as brane-world theories. There is a plethora of multi-particle BCFW shifts. We concentrate on the two simplest cases the two particle and single particle shifts. The case of the two particle shift is momentum conserving in all directions and encompasses the standard BCFW shift for both bulk and brane modes. The single particle shift, special to theories where a defect is present, includes shifts which violate momentum conservation in the directions transverse to the defect and applies only to bulk modes. The single particle shift is the one which leads to the tadpole pomeron operators of the previous section.

In section 5 we use the results of sections 3 and 4 to derive the behavior of disc scattering amplitudes under two particle momentum shifts for open and closed strings and the one particle momentum closed string shift. The pattern for the two particle shifts is the same as that for gauge theory and gravity respectively. The one particle shift leads to a behavior similar to the two particle open string shift as dictated by the Kawai–Lewellen–Tye (KLT) [23] relations. For superstring amplitudes the behavior derived is the same as the field theory analysis for gauge and gravity theories. We conclude the section with an explicit example where we compute the BCFW behavior for the superstring scattering amplitude on the disc of two level one closed string states for both the two particle and one particle shifts. We verify that indeed our pomeron analysis gives the correct behavior.

Section 6 makes an attempt to identify a field theory whose behavior under BCFW shifts is the same as that of the string theory amplitudes, therefore extending the ER regime conjecture to the case of D-branes. We do not succeed in identifying such a field theory and there are several good reasons why the conjecture might not be applicable for string amplitudes involving D-branes. The first observation is that tree level field theory diagrams describing exchange of a bulk field between D-branes correspond to loop amplitudes in the string theory side. For instance, a closed string

exchanged between D-branes is related through world-sheet duality to the annulus (one-loop) amplitude of open strings.

Another crucial point, as mentioned before, is that Regge channel factorization does not always agree with the field theory limit factorization of the amplitude. In addition, there are amplitudes whose behavior under certain BCFW shifts is not of Regge type. Nevertheless, they have the behavior predicted by the Dirac–Born–Infeld (DBI) effective field theory. However, the field theory diagrams constructed using the DBI vertices corresponding to non-Regge amplitudes, lead into disagreement with the pomeron analysis for amplitudes with Regge behavior.

These problems might have been expected since, as pointed out in [14], in a gauge theory with higher dimension operators such as $(F_{\mu\nu})^n$, $n > 2$ the nice BCFW behavior of Yang–Mills (YM) theory is spoiled. This dictates that we should take the low-energy limit of the DBI action which eliminates all the aforementioned higher dimension operators and leaves only the SYM action. Moreover, when we consider amplitudes with closed and open strings on D-branes with Regge type behavior under BCFW shifts, we find out that they cannot be reproduced by keeping closed string couplings in the DBI action. Since these couplings are proportional to Newton’s constant κ_N , they are higher dimension operators and in the spirit of the discussion above should be eliminated. We conclude that the ER regime conjecture might work only in the decoupling limit of the D-brane theory. It would be interesting to investigate if this conclusion has any implications for high energy scattering in holographic backgrounds.

2. Conformal field theory with D-branes

In this paper we will consider the bosonic string and the NS sector of the superstring. In this section we summarize for convenience the relevant notions from CFT that we will employ (see, for instance, [24, 25, 26]).

In spacetime Dp-branes are represented as static (p+1)-dimensional defects. As a result of these defects we must impose different boundary conditions on the world-sheet boundary to coordinates tangent and normal to the D-brane

$$\begin{aligned}\partial_\perp X^\alpha|_{\partial\Sigma} &= 0, \\ X^i|_{\partial\Sigma} &= 0.\end{aligned}\tag{2.1}$$

The lower case Greek indices $\alpha = 0, 1, \dots, p$ correspond to directions parallel to the D-brane while the lower Latin ones $i = p + 1, \dots, 9$ to normal coordinates. The boundary conditions (2.1) are respectively Neumann and Dirichlet.

The string vertex operators of a closed string are factorized in holomorphic and antiholomorphic parts and they take the following schematic form

$$V_{(s,\bar{s})}(z, \bar{z}) \sim \epsilon : V_{(s)}(z) :: V_{(\bar{s})}(\bar{z}) : \tag{2.2}$$

where ϵ is a polarization tensor. For the superstring s denotes the superghost charge or equivalently the picture in which the operator is in. The total superghost charge on the disk is required to be $Q_{sg} = -2$ as a consequence of superdiffeomorphism invariance.

The holomorphic parts of the vertex operators for bosonic string tachyons and level 1 states carrying momentum p are

$$\begin{aligned} V^T(z; p) &= e^{ip \cdot X(z)} , \\ V^\mu(z; p) &= \sqrt{\frac{2}{\alpha'}} \partial X^\mu(z) e^{ip \cdot X(z)} \end{aligned} \quad (2.3)$$

respectively. For example, the vertex operator for a closed string tachyon reads $V^{T_c}(z, \bar{z}; p) = V^T(z; p) \bar{V}^T(\bar{z}; p)$. Notice that the vertex operators for open string tachyons and gluons take a form similar to the above expressions.

The holomorphic parts of the vertex operators corresponding to massless states (level 1) of the closed superstring are given by

$$\begin{aligned} V_{(-1)}^\mu(z; p) &= e^{-\varphi(z)} \psi^\mu(z) e^{ip \cdot X(z)} \\ V_{(0)}^\mu(z; p) &= \sqrt{\frac{2}{\alpha'}} \left(\partial X^\mu(z) + i \frac{\alpha'}{2} p \cdot \psi(z) \psi^\mu(z) \right) e^{ip \cdot X(z)} \end{aligned} \quad (2.4)$$

in the -1 and 0 picture respectively with similar expressions for the anti-holomorphic part. As usual φ is the bosonized superconformal ghost.

The expectation values of string vertices are found using the following correlators:

$$\begin{aligned} \langle X^\mu(z) X^\nu(w) \rangle &= -\frac{\alpha'}{2} \eta^{\mu\nu} \log(z - w) , \\ \langle \psi^\mu(z) \psi^\nu(w) \rangle &= -\frac{\eta^{\mu\nu}}{z - w} , \\ \langle \varphi(z) \varphi(w) \rangle &= -\log(z - w) . \end{aligned} \quad (2.5)$$

The Minkowski metric is $\eta^{\mu\nu} = \text{diag}\{-1, +1, \dots, +1\}$. Because of the boundary conditions we have non-trivial correlators between the holomorphic and the anti-holomorphic parts

$$\begin{aligned} \langle X^\mu(z) \bar{X}^\nu(\bar{w}) \rangle &= -\frac{\alpha'}{2} D^{\mu\nu} \log(z - \bar{w}) , \\ \langle \psi^\mu(z) \bar{\psi}^\nu(\bar{w}) \rangle &= -\frac{D^{\mu\nu}}{z - \bar{w}} , \\ \langle \varphi(z) \bar{\varphi}(\bar{w}) \rangle &= -\log(z - \bar{w}) \end{aligned} \quad (2.6)$$

where D_ν^μ is a diagonal matrix with +1 for directions tangent to the world-volume and -1 for the normal directions.

$$\partial X^\mu(z) : e^{ip \cdot X(w)} : \sim -ip^\mu \frac{\alpha'}{2} \frac{e^{ip \cdot X(w)}}{z - w} , \quad (2.7)$$

$$: e^{ip_1 \cdot X(z)} : : e^{ip_2 \cdot X(w)} : \sim (z - w)^{\frac{\alpha'}{2} p_1 \cdot p_2} : e^{ip_1 \cdot X(z)} e^{ip_2 \cdot X(w)} : . \quad (2.8)$$

In some cases it will be more convenient to use the correlators on the disc [36] rather than on the half-plane:

$$\begin{aligned} \langle X^\mu(z) \bar{X}^\nu(\bar{w}) \rangle &= -\frac{\alpha'}{2} D^{\mu\nu} \log(1 - z\bar{w}) , \\ \langle \psi^\mu(z) \bar{\psi}^\nu(\bar{w}) \rangle &= i \frac{D^{\mu\nu}}{1 - z\bar{w}} . \end{aligned} \quad (2.9)$$

At this point it is useful to define two projection matrices V_ν^μ and N_ν^μ which project on the tangent and normal directions of the brane respectively and satisfy

$$D_{\mu\nu} = V_{\mu\nu} - N_{\mu\nu} , \quad \eta_{\mu\nu} = V_{\mu\nu} + N_{\mu\nu} . \quad (2.10)$$

It is important to notice that momentum is conserved only along the tangent directions. Therefore, if we have a set of strings scattering with momenta p_I^μ , momentum conservation takes the following form:

$$\sum_{i=1}^n (p_I + D p_I)^\mu = 0 . \quad (2.11)$$

Extending the definition of the fields to the whole complex plane [25] we can write all our vertices in terms of left moving string operators by making the substitutions

$$\bar{X}^\mu(\bar{z}) \rightarrow D_\nu^\mu X^\nu(\bar{z}) , \quad \bar{\psi}^\mu(\bar{z}) \rightarrow D_\nu^\mu \psi^\nu(\bar{z}) , \quad \bar{\varphi}(\bar{z}) \rightarrow \varphi(\bar{z})$$

where $z \in \mathcal{H}^+$. This way we use the standard correlators (2.5) in string amplitude computations. After these replacements we write for the case of a graviton with polarization $\epsilon_{\mu\nu}$ in the superstring

$$V_{(s,\bar{s})}(z, \bar{z}; p) = (\epsilon D)_{\mu\nu} : V_s^\mu(z; p) : : V_{\bar{s}}^\nu(\bar{z}; Dp) : \quad (2.12)$$

Similar manipulations allow us to write the open string vertex operators for Neumann and Dirichlet conditions in the -1 and 0 picture as [25]

$$\begin{aligned} V_{(-1)}^\mu(x; 2k) &= e^{-\varphi(x)} \psi^\mu(x) e^{i2k \cdot X(x)} , \\ V_{(0)}^\mu(x; 2k) &= \sqrt{\frac{2}{\alpha'}} \left(\partial X^\mu(x) + i\alpha' k \cdot \psi(x) \psi^\mu(x) \right) e^{i2k \cdot X(x)} \end{aligned} \quad (2.13)$$

with $x \in \mathbb{R}$ and the momentum k^μ restricted to be tangent to the brane. The vertex operators (2.13) have momenta $2k$ in the exponent and a slightly different form in the zero picture as compared to (2.4). This is due to the fact we use the correlators (2.5) and (2.6) or (2.9) which are for closed strings and do not take care of the double subtraction needed for normal ordering operators on the boundary of the disc

[25]⁴. This will be important later when comparing our pomeron operators with those which have appeared recently in the literature.

We have now all the ingredients to compute an amplitude between n open strings carrying momenta $k_I, I = 1, \dots, n$ and m closed strings with momenta $p_J, J = 1, \dots, m$.

$$\mathcal{A}(\{k_I\}, \{p_J\}) = \int_{D_2} \frac{d^2 z_J dx_I}{V_{CKG}} \left\langle \prod_{I=1}^n : V(x_I; k_I) : \prod_{J=1}^m : V(z_J, \bar{z}_J; p_J) : \right\rangle \quad (2.14)$$

with V_{CKG} being the volume of the conformal Killing group. The open string momenta can only be in directions tangent to the brane and momentum conservation reads⁵

$$\sum_{I=1}^n k_I + \sum_{J=1}^m V \cdot p_J = 0. \quad (2.15)$$

3. Pomeron techniques in the presence of D-branes

Pomeron vertex operator for string amplitudes first appeared in [21]. We will use their method to derive pomeron vertex operators in the presence of boundary states. We will consider the scattering of two closed string tachyons off a D-brane and we will demonstrate the Regge behavior of this amplitude. Subsequently, we will use the OPE of tachyon operators in bosonic string theory and we will compute the pomeron vertex that reproduces the aforementioned Regge behavior. The purpose of computing the Regge limit behavior for this prototype amplitude is that it will enable us to identify two qualitatively different pomeron channels due to the presence of the boundary on the world-sheet.

3.1 Pomeron operators for tachyons

We consider the amplitude on the disc D_2 of two closed string tachyons with momenta p_1 and p_2 . Using the correlators (2.9) and after dividing by V_{CKG} we obtain

$$\begin{aligned} \mathcal{A}(p_1, p_2) &\sim \int_{D_2} d^2 w \langle : V^{T_c}(w, \bar{w}; p_1) :: V^{T_c}(0, 0; p_2) : \rangle_{D_2} \\ &\sim \int_{D_2} d^2 w (1 - |w|^2)^{\alpha' s/2} |w|^{-\alpha' t/2} \end{aligned} \quad (3.1)$$

where the kinematic variables are $s = p_1 \cdot D \cdot p_1 = p_2 \cdot D \cdot p_2$ and $t = -2p_1 \cdot p_2$. As is usual in the computation of string amplitudes, we assume that the kinematic

⁴Usually in the literature one sets for simplicity $\alpha' = 2$ for both closed and open strings. We keep here α' explicitly since it is useful in order to track leading momentum contributions in our formulas.

⁵The projector V in the formula below should not be confused with a vertex operator!

variables take appropriate values where the integrals are convergent and once we are done with the integrations we can analytically continue to the physical region. For the integral in (3.1) convergence requires that $s > 0$ and $t < 0$.

Following the saddle point method of [21] we are able to identify the region of the world-sheet where the two different channels dominate:

$$\begin{aligned} \text{t-channel} & : -t \ll s \longrightarrow |w|^2 \sim -\frac{t}{s} , \\ \text{s-channel} & : s \ll -t \longrightarrow |w|^2 \sim 1 . \end{aligned} \quad (3.2)$$

The two channels are named after the subleading momentum invariant and correspond to distinct pomeron states giving the leading contribution to scattering where a set of operators is highly boosted with respect to the rest of the operators in the path integral. For scattering on the disc they correspond to two distinct factorization channels on the world-sheet: closed string (t-channel) and open string (s-channel) (see, for instance, [27]). In the ensuing OPE analysis we will use hatted symbols to indicate the momenta that are highly boosted. This will result in some kinematic invariants being larger than others, as for example in (3.2), and we will use hatted symbols for them as well.

More specifically, for amplitudes on the sphere or purely open string amplitudes on the disc, kinematic invariants built from products of boosted or unboosted momenta are unhatted (small) while those from cross-products of highly boosted momenta with unboosted ones are hatted (large). We refer the reader to [21] for further details. A subtlety is that in the presence of D-branes it is possible that kinematic invariants built only from boosted momenta are large. We shall see shortly such examples. In the present pomeron discussion this notation is not particularly useful but we will keep it in order to be in accord with the BCFW analysis which will follow later on.

3.1.1 t-channel

Let us first study the integral (3.1) in the t-channel region. The result is

$$\begin{aligned} \int_{S^2} d^2w \, e^{\alpha' \hat{s}/2 \log(1-|w|^2)} |w|^{-\alpha' t/2} & \sim \int_{S^2} d^2w \, e^{-\alpha' \hat{s}/2} |w|^2 |w|^{-\alpha' t/2} \\ & \sim \Gamma\left(1 - \frac{\alpha' t}{4}\right) \left(\frac{\alpha' \hat{s}}{2}\right)^{\frac{\alpha' t}{4}-1} . \end{aligned} \quad (3.3)$$

Notice that we are integrating on a sphere since the region $|w| \ll 1$ is far from the boundary of the disc.

The corresponding pomeron vertex operator can be extracted from the OPE

$$\begin{aligned} : e^{i\hat{p}_1 \cdot X(w, \bar{w})} : : e^{i\hat{p}_2 \cdot X(0,0)} : & \sim |w|^{-\alpha' t/2} : e^{i\hat{p}_1 \cdot X(w) + i\hat{p}_2 \cdot X(0)} : : e^{i\hat{p}_1 \cdot \bar{X}(\bar{w}) + i\hat{p}_2 \cdot \bar{X}(0)} : \\ & \sim |w|^{-\alpha' t/2} : e^{i(\hat{p}_1 + \hat{p}_2) \cdot X(0) + i w \hat{p}_1 \cdot \partial X(0)} : : e^{i(\hat{p}_1 + \hat{p}_2) \cdot \bar{X}(0) + i \bar{w} \hat{p}_1 \cdot \bar{\partial} \bar{X}(0)} : . \end{aligned} \quad (3.4)$$

As in [21] we have kept the terms subleading in w and \bar{w} . Moreover, we have normal ordered separately the holomorphic X and antiholomorphic \bar{X} fields since when this operator approaches the boundary there will be non-trivial correlators between them due to (2.9). Integrating over w we extract the pomeron vertex⁶

$$\mathcal{V}^{T_c T_c} = \Pi^c(\alpha' t) : e^{ip \cdot X(0)} (i\hat{p}_1 \cdot \partial X)^{\frac{\alpha' t}{4}-1} : : e^{ip \cdot \bar{X}(0)} (i\hat{p}_1 \cdot \bar{\partial} \bar{X})^{\frac{\alpha' t}{4}-1} : \quad (3.5)$$

where $p = \hat{p}_1 + \hat{p}_2$ and the closed pomeron propagator reads

$$\Pi^c(\alpha' t) = \frac{\Gamma(1 - \frac{\alpha' t}{4})}{\Gamma(\frac{\alpha' t}{4})} e^{i\pi(1 - \frac{\alpha' t}{4})}. \quad (3.6)$$

Actually, to be closer to the spirit of [21] we should expand the OPE symmetrically in the positions of the two operators, therefore obtaining

$$\mathcal{V}^{T_c T_c} = \Pi^c(\alpha' t) : e^{ip \cdot X(0)} (iQ \cdot \partial X)^{\frac{\alpha' t}{4}-1} : : e^{ip \cdot \bar{X}(0)} (iQ \cdot \bar{\partial} \bar{X})^{\frac{\alpha' t}{4}-1} : \quad (3.7)$$

where $Q = \frac{\hat{p}_1 - \hat{p}_2}{2}$. This operator satisfies the physical state conditions of the Virasoro algebra

$$\begin{aligned} L_0 \mathcal{V}^{T_c T_c} &= \frac{\alpha'}{4} p^2 + N - 1 = \frac{\alpha'}{4} (\hat{p}_1 + \hat{p}_2)^2 + \left(\frac{\alpha'}{4} t - 1 \right) - 1 = 0, \\ L_1 \mathcal{V}^{T_c T_c} &\sim Q \cdot p = 0. \end{aligned} \quad (3.8)$$

Taking the expectation value of this operator on the disc D_2 and using momentum conservation along the D-brane $(p + Dp)^\mu = 0$, yields

$$\begin{aligned} \langle \mathcal{V}^{T_c T_c}(0) \rangle_{D_2} &\sim \Pi^c(\alpha' t) \left(\frac{\alpha' t}{4} - 1 \right)! \left(-\frac{\alpha'}{2} Q \cdot D \cdot Q \right)^{\frac{\alpha' t}{4}-1} \\ &\sim \Gamma\left(1 - \frac{\alpha' t}{4}\right) \left(\frac{\alpha' \hat{s}}{2} \right)^{\frac{\alpha' t}{4}-1} \end{aligned} \quad (3.9)$$

which indeed agrees with (3.3). We have used that $Q \cdot D \cdot Q = s - \frac{t}{4} + \frac{m_{T_c}^2}{2}$, where $m_{T_c}^2 = -4/\alpha'$ is the tachyon squared-mass, and so $Q \cdot D \cdot Q \sim s$ in the kinematic regime under consideration.

3.1.2 s-channel

Now let us proceed with the s-channel region. In this case (3.1) becomes

$$\begin{aligned} \int_{D^2} d^2 w e^{-\alpha' \hat{t}/4 \log |w|^2} (1 - |w|^2)^{\alpha' s/2} &\sim \int_{D^2} d^2 w e^{-\alpha' \hat{t}/4 (|w|^2 - 1)} (1 - |w|^2)^{\alpha' s/2} \\ &\sim \Gamma\left(\frac{\alpha' s}{2} + 1\right) \left(-\frac{\alpha' \hat{t}}{4}\right)^{-\frac{\alpha' s}{2}-1}. \end{aligned} \quad (3.10)$$

⁶Notice that compared to the pomeron vertex operator of (3.6) in [21] we have an extra phase. This is in agreement with (3.20) and (3.21) of the same paper.

We have dropped the contribution of an incomplete gamma function which goes to zero for large $\alpha't/4$.

Notice that this world-sheet region dominates when a single string vertex operator approaches the disc boundary which we choose to be the operator with momentum \hat{p}_1 . The OPE we need to consider this time is between the holomorphic and anti-holomorphic pieces of the corresponding vertex operator near the disc boundary

$$\begin{aligned} : e^{i\hat{p}_1 \cdot X(w)} :: e^{i\hat{p}_1 \cdot \bar{X}(\bar{w})} : &\sim (w - \bar{w})^{\alpha's/2} : e^{i\hat{p}_1 \cdot X(w)} e^{iD \cdot \hat{p}_1 \cdot X(\bar{w})} : \\ &\sim (2iy)^{\alpha's/2} : e^{i(\hat{p}_1 + D \cdot \hat{p}_1) \cdot X(x) + iy\hat{p}_1 \cdot (\partial X - D \cdot \partial X)} : \end{aligned} \quad (3.11)$$

where we used this time correlators on the upper half-plane and the parametrization $w = x + iy$ with $y = 0$ being the boundary. Performing the integral over y and using the identity $\hat{p} + D \cdot \hat{p} = 2V \cdot \hat{p}$ results in

$$\mathcal{V}^{T_c} = \Pi^o(\alpha's) (i\hat{p}_1 \cdot N \cdot \partial X)^{-1-\alpha's/2} e^{i(2V \cdot \hat{p}_1) \cdot X} \quad (3.12)$$

or, in the symmetric OPE expansion,

$$\mathcal{V}^{T_c} = \Pi^o(\alpha's) (iQ \cdot \partial X)^{-1-\alpha's/2} e^{i(2k) \cdot X} \quad (3.13)$$

where $2k = \hat{p}_1 + D \cdot \hat{p}_1 = 2V \cdot \hat{p}_1$ and $2Q = \hat{p}_1 - D \cdot \hat{p}_1 = 2N \cdot \hat{p}_1$ and the pomeron propagator is

$$\Pi^o(\alpha's) = \Gamma\left(\frac{\alpha's}{2} + 1\right) . \quad (3.14)$$

The subscript indicates that the pomeron vertex corresponds to the factorization of the amplitude into a closed string tachyon tadpole on the D-brane connected via the pomeron with the rest of the operators in the path integral. Such a behavior is possible due to the non-conservation of momentum normal to the D-brane and is one of the novelties of the present analysis.

This operator is an open string pomeron vertex. We can verify that the physical state conditions are satisfied⁷

$$\begin{aligned} L_0 \mathcal{V}^{T_c} &= \frac{\alpha'}{4} (2k)^2 + N - 1 = \alpha' (V \hat{p}_1)^2 + \left(-1 - \frac{\alpha'}{2} s\right) - 1 = 0 , \\ L_1 \mathcal{V}^{T_c} &\sim Q \cdot k = N \cdot \hat{p}_1 \cdot V \cdot \hat{p}_1 = 0 . \end{aligned} \quad (3.15)$$

To compare with the corresponding expression in (3.10) it is instructive to compute the amplitude in the symmetric manner indicated in [21]. This is the symmetric picture of pomeron exchange which separates the boosted particles into two sets and

⁷The level N in the first formula below should not be confused with the projector N in the second formula!

uses the pomeron vertex⁸ to connect the individual diagrams. We can indeed compute the amplitude due to pomeron exchange between two oppositely boosted closed string tachyons with momenta \hat{p}_1 and \hat{p}_2

$$\begin{aligned}
\mathcal{A}(\hat{p}_1, \hat{p}_2) &\sim \langle V_1^{T_c} \mathcal{V}^{T_c} \rangle_{D_2} \Pi^o(\alpha' s) \langle V_2^{T_c} \mathcal{V}^{T_c} \rangle_{D_2} \\
&\sim \left(\sqrt{\frac{\alpha'}{2}} N \cdot \hat{p}_1 \right)^{-1-\alpha' s/2} \cdot \Pi^o(\alpha' s) \left(\sqrt{\frac{\alpha'}{2}} N \cdot \hat{p}_2 \right)^{-1-\alpha' s/2} \\
&\sim \left(-\frac{\alpha' \hat{t}}{4} \right)^{-1-\alpha' s/2} \Gamma \left(1 + \frac{\alpha' s}{2} \right)
\end{aligned} \tag{3.16}$$

where we have suppressed the corresponding ghost contributions which are needed to fix the individual CKG of the two discs. The factors $\frac{\alpha'}{2}$ in the computation of each disc amplitude appear due to the normalization of the pomeron operators, as can be derived by equation (3.12) of [21] for the open pomeron and the contractions using (2.5) and (2.6). Moreover we have used the fact that in this regime we have $\hat{t} = -2\hat{p}_1 \cdot \hat{p}_2 \sim -2\hat{p}_1 \cdot N \cdot \hat{p}_2$, since $-2\hat{p}_1 \cdot V \cdot \hat{p}_2 \ll \hat{p}_1 \cdot N \cdot \hat{p}_2$ in the kinematic regime under consideration and the expression in (3.16) assumes contractions of the tensors between the amplitude of the one disc and the other⁹.

We can also repeat the t-channel computation in this symmetric formalism

$$A \sim \langle \mathcal{V}^{T_c} \rangle_{D_2} \Pi^c(\alpha' t) \langle \mathcal{V}^{T_c} V_1^T V_2^T \rangle_{S^2} \tag{3.17}$$

A short computation of the two individual diagrams and contraction using the propagator results in the expression (3.9).

At this point we should make two comments. First, the reason we have insisted on this symmetric formulation is that although in the present paper we will discuss two-particle BCFW deformation of the amplitudes there exist more general ones involving three or more deformed momenta. These have appeared in the literature in various works [6, 28]. The behavior of amplitudes under more general deformations can be studied using this symmetric formalism by grouping two, three or more particles on the left part of the amplitude and the rest on the right one. We will say a bit more on this point in the conclusions.

Second, note that for the corresponding superstring scattering of level one states, that is the graviton, the dilaton and the Kalb–Ramond field [25], the two pomeron channels correspond to the field theory channels which reproduce the expected behavior from the gravity plus DBI actions (see for example [29]). From a technical

⁸Notice that in this context the pomeron vertex (3.13) appears stripped of its polarization vector iQ^μ , much like in field theory the intermediate gauge bosons polarizations are substituted with their propagator.

⁹A more elegant way to put it is the $+/-$ plane formalism of [21] but we will be a bit sloppy and make our notation more compact.

point of view this comes from the fact that in these amplitudes there is an overall beta function expression which multiplies an expression which does not depend on α' and therefore it goes into itself in the field theory limit $\alpha' \rightarrow 0$. The field theory poles come from massless poles of the beta function expression. When we take the opposite limit, i.e. Regge one, the pomerons effectively average over the zeros and poles of the beta function which lie on the real axis of the kinematic variable [30]. It is therefore no surprise that the structure and factorization channels of the two different limits agree. But it is highly non-trivial in more complicated amplitudes where this simple structure is not expected to persist.

3.1.3 Application 1: closed string tachyon scattering with three open strings

In this section we would like to demonstrate how the pomeron operators can be used to extract the Regge behavior for amplitudes involving both closed and open string states. Our results will be relevant for the comparison with the field theory expectations in the spirit of [14]. In principle we should consider an amplitude closely related to (3.1). It has been shown in [23] that closed string amplitudes can be constructed by gluing of open string amplitudes. The four open string case on the disc can be used in similar manner to construct [24] the two closed and the two open - one closed string amplitudes based on the identification (2.12). Recently this idea has been pushed even further to demonstrate in full generality the relation of pure open string amplitudes to amplitudes with both open and closed strings [31].

In the previous section we studied the two closed tachyons amplitude. The next case one could try is amplitude of two open tachyons with momenta k_1, k_2 and one closed tachyon with momentum p

$$\mathcal{A}(p, k_1, k_2) \sim \frac{\Gamma(1 - \alpha't)}{\Gamma(1 - \frac{\alpha'}{2}t)^2} . \quad (3.18)$$

Due to momentum conservation there is only one kinematic variable $t = -2k_1 \cdot k_2 = 2k_1 \cdot p + \frac{2}{\alpha'} = 2k_2 \cdot p + \frac{2}{\alpha'}$ and the amplitude does not exhibit Regge type behavior. One can indeed see that its large t behavior is not a pomeron like one. This will actually be important for our discussion in section 6.

The next case to consider is the three open and one closed tachyon amplitude. The corresponding amplitude in the superstring for three scalars and one graviton has been computed in [26]. The bosonic tachyon case can be extracted easily

$$\begin{aligned} \mathcal{A}(p, k_1, k_2, k_3) &\sim \int_{\mathcal{H}^+} d^2z |1 - z|^{\alpha'(s+u)/2-2} |z|^{\alpha'(t+u)/2-2} (z - \bar{z})^{-\alpha'(s+t+u)/2+1} \\ &\sim \frac{\Gamma(\frac{1}{2} - \alpha'\frac{s}{4}) \Gamma(\frac{1}{2} - \alpha'\frac{t}{4}) \Gamma(\frac{1}{2} - \alpha'\frac{u}{4}) \Gamma(1 - \alpha'\frac{s+t+u}{4})}{\Gamma(1 - \alpha'\frac{s+u}{4}) \Gamma(1 - \alpha'\frac{s+t}{4}) \Gamma(1 - \alpha'\frac{t+u}{4})} \end{aligned} \quad (3.19)$$

where the kinematic variables are defined as

$$s = -4k_1 \cdot k_2, \quad t = -4k_1 \cdot k_3, \quad u = -4k_2 \cdot k_3 \quad (3.20)$$

and they satisfy

$$2(V \cdot p)^2 + s + t + u = \frac{6}{\alpha'} . \quad (3.21)$$

First lets try to find a Regge regime of the s-channel type as in (3.10). This corresponds to the factorization of the three open string states on a disc and the closed string state on another disc connected with a strip (open string state). But as in the case of (3.18) there is no Regge type behavior with such a factorization. This is due to momentum conservation

$$V \cdot p + k_1 + k_2 + k_3 = 0 \quad (3.22)$$

which dictates that if we highly boost the three open string states in one direction the closed string is boosted as well and there is no pomeron exchange among them.

In the supersymmetric computation of [26] the amplitude is the product of a an expression similar to (3.19) and a kinematic prefactor which depends on the momenta and polarizations of the scattered particles. In that case the amplitude has a factor $\Gamma(-\alpha' \frac{s+t+u}{4})$ whose massless pole give rise to the field theory limit of the amplitude. From the discussion above it should be clear that there is no Regge type behavior which leads to the same factorization on the world-sheet as the field theory limit does. This will be important in our discussion in section 6.

Instead, we can choose to boost only two open string tachyons, say those with momenta k_1 and k_2 , which corresponds to the kinematic region

$$(V \cdot p)^2, s \ll \hat{t}, \hat{u} . \quad (3.23)$$

In this limit

$$\frac{\Gamma(\frac{1}{2} - \alpha' \frac{u}{4})}{\Gamma(1 - \alpha' \frac{s+u}{4})} \sim \left(\frac{\alpha' \hat{u}}{4}\right)^{\frac{\alpha' s}{4} - \frac{1}{2}} e^{-\frac{\alpha' s}{4}} \quad (3.24)$$

and similarly for the ratio of the other two Gamma function involving \hat{t} and s . Since $\hat{t} \sim -\hat{u}$ due to (3.21) we finally obtain

$$\mathcal{A}(p, \hat{k}_1, \hat{k}_2, k_3) \sim e^{-\frac{\alpha' s}{2}} \left(\frac{\alpha' \hat{u}}{4}\right)^{\frac{\alpha' s}{2} - 1} \Gamma\left(\frac{2 - \alpha' s}{4}\right) \frac{\Gamma\left(\frac{\alpha'(V \cdot p)^2 - 1}{2}\right)}{\Gamma\left(\frac{\alpha'(V \cdot p)^2 - 1}{2} + \frac{\alpha' s}{4}\right)} . \quad (3.25)$$

Let us now see how we can reproduce this behavior in terms of the pomeron. Using the open pomeron vertex of [18]¹⁰ we arrive at the expression

$$\mathcal{A}(p, \hat{k}_1, \hat{k}_2, k_3) \sim \int_0^\infty dx \langle (c(x) + c(-x)) c(i) \tilde{c}(-i) \rangle \times \Gamma\left(1 - \frac{\alpha'}{2}s\right) \times \left\langle \left((\hat{k}_1 - \hat{k}_2) \cdot \partial X \right)^{\frac{\alpha'}{2}s-1} e^{2i(\hat{k}_1 + \hat{k}_2) \cdot X}(x) e^{2ik_3 \cdot X}(-x) e^{ip \cdot X}(i) e^{iDp \cdot X}(-i) \right\rangle. \quad (3.26)$$

This amplitude contains the open pomeron corresponding to the highly boosted strings with momenta \hat{k}_1 and \hat{k}_2 and the unboosted open and closed tachyons with momenta k_3 and p respectively. We have inserted the open pomeron at x on the boundary, the open tachyon at $-x$ and the closed tachyon at $(z, \bar{z}) = (i, -i)$.

After performing the operator contractions we obtain

$$\mathcal{A}(p, \hat{k}_1, \hat{k}_2, k_3) \sim (\alpha'(\hat{u} - \hat{t}))^{\frac{\alpha'}{2}s-1} \Gamma\left(1 - \frac{\alpha'}{2}s\right) \mathcal{I}(s, V \cdot p) \quad (3.27)$$

where the integral

$$\mathcal{I}(s, V \cdot p) = \int_0^\infty dy (y-1)^{\alpha's/2-1} y^{(\alpha'(V \cdot p)^2-3)/2} (1+y)^{-\alpha'(V \cdot p)^2+2-\alpha's/2} = \frac{\Gamma^2\left(\frac{\alpha'}{2}(V \cdot p)^2 - \frac{1}{2}\right)}{\Gamma(\alpha'(V \cdot p)^2 - 1)} {}_2F_1\left(1 - \frac{\alpha'}{2}s, \frac{\alpha'}{2}(V \cdot p)^2 - \frac{1}{2}; \alpha'(V \cdot p)^2 - 1; 2\right) \quad (3.28)$$

can be found in [32]. Using the identities [33]

$${}_2F_1\left(1 - \frac{\alpha'}{2}s, \frac{\alpha'}{2}(V \cdot p)^2 - \frac{1}{2}; \alpha'(V \cdot p)^2 - 1; 2\right) = (-1)^{\alpha's/4-1/2} {}_2F_1\left(\frac{\alpha'}{2}s - \frac{1}{2}, \frac{\alpha'}{2}(V \cdot p)^2 + \frac{\alpha's}{4} - 1; \frac{\alpha'}{2}(V \cdot p)^2; 1\right) \quad (3.29)$$

and

$${}_2F_1(a, b; c; 1) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} \quad (3.30)$$

we can indeed match this result with the expression in (3.25). This verifies our formulation and moreover demonstrates that Regge behavior is indeed distinct from the field theory limit making the conjecture of eikonal Regge region of [18] non-trivial.

We would like to make a final comment before moving on to the next section. In (3.26) we chose to boost the momenta \hat{k}_1, \hat{k}_2 in order to compute the amplitude. We therefore used the pomeron vertex form for the OPE of two open string tachyons. We could have chosen to work with the OPE of the closed string with one open string,

¹⁰Notice that $\alpha'_{here} = 2\alpha'_{there}$.

corresponding to a boost of \hat{p} and, say, \hat{k}_3 . This leads to a new pomeron, $\mathcal{V}^{T_c T_o}$, which describes the mixing of one open with one closed string tachyon. The OPE is in principle more complicated since it has two singularities to integrate over, one from the closed string operator approaching the open string and a second one from the self-interaction of the closed string operator with its image when it approaches the boundary. Nevertheless, if $\alpha' \hat{p} \cdot D \cdot \hat{p} = \alpha' \hat{p} \cdot V \cdot \hat{p} = 4$, that is when the closed string has only world-volume momenta, the computation using the OPE is rather easy and it leads to the pomeron operator discussed in appendix B. One then proceeds with the three point amplitude of this pomeron operator and the other two open string tachyons on the disc reproducing once more the limiting behavior (3.25).

3.1.4 Application 2: ultrarelativistic D-brane scattering

In this section we will compute the absorptive part of the 1-loop partition function of two ultrarelativistic branes using pomeron operators. Our result will be compared to the result of [22].

We would like to consider scattering of D-branes which move at high relative velocities very close to the speed of light. The scattering of two such objects is dictated, in the lowest order in perturbation theory, by computing the one-loop partition function for the open strings stretching between them. Through world-sheet duality this is related to the emission of a closed string state from one brane and absorption by the other. We will concentrate to the bosonic case although the supersymmetric one is totally analogous.

Lets begin by defining the setup and a few useful quantities. For a Dp -brane moving in a d -dimensional spacetime with velocity \vec{V}_1 we have the momentum vector

$$P_1^\mu = M_p \left(\frac{1}{\sqrt{1 - V_1^2}}, \frac{\vec{V}_1}{\sqrt{1 - V_1^2}} \right) \quad (3.31)$$

where M_p is the mass of the D-brane. Assume that the two D-branes have velocities very close to the speed of light, $|\vec{V}_1| = |\vec{V}_2| = 1 - q$, $q \ll 1$, and opposite to each other $\vec{V}_1 = -\vec{V}_2$. The kinematic invariant describing the scattering process is

$$s = -P_1 \cdot P_2 = M_p^2 \left(\frac{1 - V_1 V_2}{\sqrt{1 - V_1^2} \sqrt{1 - V_2^2}} \right) \simeq M_p^2 \frac{1}{q}. \quad (3.32)$$

The D-branes have world-volume coordinates x^α , $\alpha = 0, \dots, p$ with Neumann boundary conditions while the transverse directions x^i , $i = p + 1, \dots, d - 1$ have Dirichlet. For simplicity assume that the two D-branes move oppositely along the direction x^{d-1} and are separated by a distance b along the direction $x_\perp = x^{d-2}$. We define $x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^{d-1})$ light cone coordinates. The first brane is boosted along the x^+ direction and the second one along x^- .

The pomeron computation for this scattering takes the form¹¹

$$\begin{aligned} \mathcal{A} &\sim \langle B, V_1 | \Delta | B, V_2 \rangle \simeq \\ &\simeq_{(D_2, V_1)} \langle \mathcal{V}^{T_c T_c}(0) \rangle \Pi^c(\alpha' \vec{k}_\perp^2) \langle \mathcal{V}^{T_c T_c}(0) \rangle_{(D_2, V_2)} \end{aligned} \quad (3.33)$$

where Δ is the closed string propagator and $|B, V\rangle$ is the boundary state for a brane moving with velocity V . In the second line we use the CFT approach and consider the boundary conditions on the disc D_2 for moving branes with velocities V_1 and V_2 , while \vec{k}_\perp is the pomeron momentum which is transverse to the D-branes and to their line of motion, i.e. it is along the x_\perp direction. In the notation of subsection 3.1 we have $-t + 8/\alpha' = p^2 = \vec{k}_\perp^2$. The pomeron propagator (3.6) takes explicitly the form

$$\Pi^c(\alpha' \vec{k}_\perp^2) = \frac{\Gamma\left(-1 + \frac{\alpha' \vec{k}_\perp^2}{4}\right)}{\Gamma\left(2 - \frac{\alpha' \vec{k}_\perp^2}{4}\right)} e^{i\pi\left(-1 + \frac{\alpha' \vec{k}_\perp^2}{4}\right)}. \quad (3.34)$$

We use now the one-point function of the pomeron operators (3.7) on the D-brane to compute

$$\langle \mathcal{V}_{T_c T_c}(0) \rangle_{(D_2, V_1)} \sim \left\langle : \left(\frac{\partial X^+}{\sqrt{\alpha'}} \right)^{1-\alpha' \vec{k}_\perp^2/4} e^{ip \cdot X} : : \left(\frac{\bar{\partial} \bar{X}^+}{\sqrt{\alpha'}} \right)^{1-\alpha' \vec{k}_\perp^2/4} e^{ip \cdot \bar{X}} : \right\rangle_{(D_2, V_1)}. \quad (3.35)$$

We have assumed that V_1 is along x^+ and inserted the appropriate pomeron vertex normalized as in [21]. The correlator is computed on the boundary state of a boosted D-brane [34]. One can compute the matrix $M(V)$ defined in [34], which describes the gluing of holomorphic and antiholomorphic fields on the moving D-brane, in the ultrarelativistic limit. It turns out that the leading correlator, on the disc, is

$$X^+(z) \bar{X}^+(\bar{w}) \sim -\frac{\alpha'}{2} \frac{1}{q} \ln(1 - z\bar{w}). \quad (3.36)$$

The brane glues together the holomorphic X^+ and antiholomorphic \bar{X}^+ string coordinates. Notice that for light cone coordinates the only non-vanishing correlators between holomorphic fields is $\langle X^+(z) X^-(w) \rangle$ and similarly for the antiholomorphic fields.

We will also need the boundary state normalization from [34]

$$\mathcal{N}_V = \langle 1 \rangle_{(D_2, V)} \simeq (2\pi\sqrt{\alpha'})^{d_\perp} \sqrt{1 - V^2} \delta(x^{d-1} - x^0 V - y^{d-1}) \prod_{i \neq d-1} \delta(x^i - y^i) \quad (3.37)$$

where we have defined $d_\perp = d - p - 2$, ignored factors of π and kept only factors depending on α' and V . The delta function describes the motion of the brane in

¹¹Notice that here $\mathcal{V}^{T_c T_c}$ is the stripped pomeron vertex, see also footnote 7.

spacetime and depends on the zero modes x^i , $i = p + 1, \dots, d - 1$ and the initial position y^i . The final result, after combining all the ingredients and using Euler's reflection formula $\Gamma(x)\Gamma(1-x) = \frac{\pi}{\sin \pi x}$, takes the form

$$\mathcal{A} \sim V_p \frac{(2\pi\sqrt{\alpha'})^{2d_\perp}}{|V_1 - V_2|} \sqrt{1 - V_1^2} \sqrt{1 - V_2^2} \left(\frac{s}{M_p^2}\right)^{2 - \alpha' \vec{k}_\perp^2/2} F\left(\frac{\alpha' \vec{k}_\perp^2}{2}\right) \quad (3.38)$$

where

$$F\left(\frac{\alpha' \vec{k}_\perp^2}{2}\right) = -\frac{e^{i\pi\left(-1 + \frac{\alpha' \vec{k}_\perp^2}{4}\right)}}{\sin \pi\left(-1 + \frac{\alpha' \vec{k}_\perp^2}{4}\right)}. \quad (3.39)$$

The factor $|V_1 - V_2|$ in the denominator of (3.38) comes from integration over the zero modes of the two delta functions of the boundary states in (3.37). In the ultrarelativistic limit under consideration we have

$$\frac{\sqrt{1 - V_1^2} \sqrt{1 - V_2^2}}{|V_1 - V_2|} \simeq q \simeq \frac{M_p^2}{s}. \quad (3.40)$$

One can alternatively boost the branes back to their approximate rest frames extracting the boost parameter (s/s_0) where $s_0 = 2M_p^2$ is the rest frame center of mass energy. Then, as explained in [21], the equation (3.33) becomes

$$\mathcal{A} \sim \mathcal{N}_{V_1} \mathcal{N}_{V_2} \left(\frac{s}{s_0}\right)^{2 - \alpha' \vec{k}_\perp^2/2} \langle \mathcal{V}^{T_c T_c}(0) \rangle_{(D_2, V_1)} \Pi^c(\alpha' \vec{k}_\perp^2) \langle \mathcal{V}^{T_c T_c}(0) \rangle_{(D_2, V_2)}. \quad (3.41)$$

Notice that compared to (3.20) of [21], the normalization factors (3.37) need to be taken into account in order to derive the correct result.

Let us finally compute absorptive part of the scattering amplitude, $\text{Im}(\delta)$, in order to compare with the corresponding expression in [22]. This is given by the Fourier transform in position space of the imaginary part of the expression in (3.38). Notice that from (3.39) $\text{Im}(F) = -1$. We can easily show, as in [21] and using (3.40), that

$$\begin{aligned} \text{Im}(\delta) &\simeq \int d^{d_\perp} k_\perp e^{i\vec{k}_\perp \cdot \vec{x}_\perp} \text{Im}(\mathcal{A}) \simeq \\ &\simeq V_p (2\pi\sqrt{\alpha'})^{2d_\perp} \left(\frac{s}{M_p^2}\right) \int d^{d_\perp} k_\perp e^{i\vec{k}_\perp \cdot \vec{x}_\perp} e^{-\frac{\alpha'}{2} \vec{k}_\perp^2 \log\left(\frac{s}{M_p^2}\right)} \simeq \\ &\simeq V_p \frac{s}{M_p^2} \left[\log\left(\frac{s}{M_p^2}\right)\right]^{-\frac{d_\perp}{2}} e^{-\frac{b^2}{2\alpha' \log\left(\frac{s}{M_p^2}\right)}} \end{aligned} \quad (3.42)$$

with the impact parameter $x_\perp = b$. This is indeed the result of [22] for $d = 10$. The scattering has the diffusive form in transverse position space typical of Regge scattering. The characteristic size, as seen from an observer placed on the other fast moving brane, grows with the center of mass energy as $b^2 \sim \alpha' \log\left(\frac{s}{M_p^2}\right)$.

3.2 Pomeron operators for level one states

In this subsection we give the pomerons for level one states in the bosonic and superstring theories. These operators can be easily derived using similar OPE techniques as we used earlier for the tachyons. In all cases we kept the leading and sub-leading terms in the OPEs before integrating. Some of the results are given in [18, 19] and we list them along with our results.

3.2.1 Bosonic pomerons

The bosonic operators are as follows. The one involving two gauge bosons (or transverse scalars) reads

$$\mathcal{V}_{bos}^{AA} \sim \hat{\epsilon}_1^\mu \hat{\epsilon}_2^\nu e^{2ik \cdot X} \Gamma(2\alpha' k_1 \cdot k_2) (-2iQ \cdot \partial X)^{-2\alpha' k_1 \cdot k_2} \left(\frac{-2iQ \cdot \partial X}{2\alpha' k_1 \cdot k_2 - 1} (2\alpha' k_\mu k_\nu - \eta_{\mu\nu}) + 2i(\partial X_\mu k_\nu - \partial X_\nu k_\mu) \right) \quad (3.43)$$

where $k = \hat{k}_1 + \hat{k}_2 = k_1 + k_2$ and $2Q = \hat{k}_1 - \hat{k}_2$.

The pomeron with two massless closed strings (gravitons, antisymmetric tensors or dilatons) in the bulk is

$$\mathcal{V}_{bos}^{GG} \sim -\hat{\epsilon}_1^{\mu\bar{\mu}} \hat{\epsilon}_2^{\nu\bar{\nu}} e^{ip \cdot X} \Pi^c(\alpha' p_1 \cdot p_2) (-Q \cdot \partial X \bar{Q} \cdot \bar{\partial} X)^{-\frac{\alpha'}{2} p_1 \cdot p_2} \quad (3.44)$$

$$\left[(Q \cdot \partial X) P_{\mu\nu} - K_{\mu\nu} \left(\frac{\alpha'}{2} p_1 \cdot p_2 - 1 \right) \right] \left[(Q \cdot \bar{\partial} X) \bar{P}_{\bar{\mu}\bar{\nu}} - \bar{K}_{\bar{\mu}\bar{\nu}} \left(\frac{\alpha'}{2} p_1 \cdot p_2 - 1 \right) \right]$$

where $p = \hat{p}_1 + \hat{p}_2$, $2Q = \hat{p}_1 - \hat{p}_2$ and

$$P_{\mu\nu} = \frac{\alpha'}{2} p_\mu p_\nu - \eta_{\mu\nu}, \quad K_{\mu\nu} = \partial X_\mu p_\nu - \partial X_\nu p_\mu, \quad (3.45)$$

$$\Pi^c(\alpha' p_1 \cdot p_2) = \frac{\Gamma\left(\frac{\alpha'}{2} p_1 \cdot p_2 - 1\right)}{\Gamma\left(2 - \frac{\alpha'}{2} p_1 \cdot p_2\right)} e^{i\pi\left(-1 + \frac{\alpha'}{2} p_1 \cdot p_2\right)} \quad (3.46)$$

The form of the operator is determined either from the previous pomeron vertex by KLT or explicitly by computing the OPE of two operators at positions w_1 and w_2 and then integrating over their relative position $(w_1 - w_2)/2$ as in the tachyon case (3.4)-(3.5) (see Appendix A for the explicit derivation). We use barred and unbarred indices to distinguish the two independent Lorentz spin symmetries in the spirit of [14].

Finally, for the pomeron involving a massless closed string state on the disk (tadpole) we can use the doubling trick [24] which allows to construct a graviton vertex operator as a direct product of two gauge boson operators. The polarization is given by

$$\epsilon_{\mu\bar{\lambda}} D_\nu^{\bar{\lambda}} = (\epsilon D)_{\mu\nu} = \epsilon_\mu \otimes \epsilon'_\nu. \quad (3.47)$$

Of course the pomeron operator can be constructed directly using similar steps as in appendix A. The corresponding vertex is given by the expression

$$\begin{aligned} \mathcal{V}_{bos}^G \sim \hat{\epsilon}^{\mu\bar{\nu}} e^{2iV \cdot \hat{p} \cdot X} \Gamma \left(-1 + \frac{\alpha'}{2} \hat{p} \cdot D \cdot \hat{p} \right) (-iQ \cdot \partial X)^{-\frac{\alpha'}{2} \hat{p} \cdot D \cdot \hat{p}} \left[(-iQ \cdot \partial X)(D \cdot P)_{\mu\bar{\nu}} \right. \\ \left. + 2i \left(\partial X_\mu (V \cdot \hat{p})_{\bar{\nu}} - \partial (D \cdot X)_{\bar{\nu}} (V \cdot \hat{p})_\mu \right) \left(-1 + \frac{\alpha'}{2} \hat{p} \cdot D \cdot \hat{p} \right) \right] \end{aligned} \quad (3.48)$$

where $2Q = \hat{p} - D \cdot \hat{p} = 2N \cdot \hat{p}$, we have used the fact that $\hat{\epsilon}^{\mu\bar{\nu}} D \cdot \hat{p}_\nu = 2\hat{\epsilon}^{\mu\nu} (V \cdot \hat{p})_{\bar{\nu}}$ due to the physical state condition and defined $P_{\mu\nu} = -\eta_{\mu\nu} + 2\alpha' (V \cdot \hat{p})_\mu (V \cdot \hat{p})_\nu$. Notice the difference of the prefactors compared to the analogous result for the Type I superstring in the zero picture in [18]. Our prefactors have tachyon poles as expected for the bosonic theory.

3.2.2 Superstring pomerons

For the type II superstring with D-branes, which is the case of interest in the present study, the zero picture results can be found in [18].

For two gauge bosons both in the -1 picture, we derive the following

$$\begin{aligned} \mathcal{V}_{susy}^{A(-1)A(-1)} \sim \hat{\epsilon}_1^\mu \hat{\epsilon}_2^\nu e^{-2\varphi} e^{2ik \cdot X} \Gamma(2\alpha' k_1 \cdot k_2 - 1) (-2iQ \cdot \partial X)^{-2\alpha' k_1 \cdot k_2} \\ \left(\eta_{\mu\nu} (-2iQ \cdot \partial X) - \psi_\mu \psi_\nu (2\alpha' k_1 \cdot k_2 - 1) \right) \end{aligned} \quad (3.49)$$

where we have designated the superghost charge of the operator. As in the previous examples we have defined $k = \hat{k}_1 + \hat{k}_2 = k_1 + k_2$ and $2Q = \hat{k}_1 - \hat{k}_2$.

For two massless closed strings we choose an asymmetric picture for the graviton operators. The reason for this is that if we wish to insert them on the disc we should not exceed the total superghost charge of -2 , otherwise we will have to insert picture changing operators. We will compute the OPE of $V_{(0,-1)}$ with $V_{(-1,0)}$ by calculating separately the holomorphic OPE and using KLT to derive the closed string OPE. The holomorphic OPE we wish to compute is

$$V_{(0)}(y) V_{(-1)}(-y) \sim \hat{\epsilon}_{1;\mu} \left(\partial X^\mu + i\alpha' (\hat{k}_1 \cdot \psi) \psi^\mu \right) e^{2i\hat{k}_1 \cdot X}(y) \hat{\epsilon}_{2;\nu} \psi^\nu e^{-\varphi} e^{2i\hat{k}_2 \cdot X}(-y). \quad (3.50)$$

The final result (see appendix A for some details) after the integration over y is

$$\begin{aligned} \mathcal{V}_{susy}^{A(-1)A(0)} \sim \hat{\epsilon}_{1;\mu} \hat{\epsilon}_{2;\nu} e^{-\varphi} e^{2ik \cdot X} \Gamma(2\alpha' k_1 \cdot k_2 - 1) (Q \cdot \partial X)^{-1-2\alpha' k_1 \cdot k_2} \\ \left[\eta^{\mu\nu} (Q \cdot \partial X) (\hat{k}_1 \cdot \psi) + (2\alpha' k_1 \cdot k_2 - 1) \times \right. \\ \left. \left(-2(\hat{k}_1 \cdot \psi) \psi^\mu \psi^\nu + \eta^{\mu\nu} \left((\hat{k}_1 \cdot \psi) \dot{\phi} + (\hat{k}_1 \cdot \dot{\psi}) \right) \right) - (Q \cdot \partial X) (\psi^\mu k^\nu - \psi^\nu k^\mu) \right]. \end{aligned} \quad (3.51)$$

Now we can take the product of two such pomeron operators, one of which corresponds to the OPE of the holomorphic parts of the operators and the other

to the antiholomorphic, to obtain the closed string pomeron. One only needs to make the substitutions $2k_i \rightarrow p_i$ for the holomorphic part, $2k_i \rightarrow D \cdot p_i$ for the antiholomorphic and to use the polarizations map $\hat{\epsilon}_{i;\mu} \times \hat{\epsilon}_{i;\nu} \rightarrow (\epsilon D)_{\mu\nu} = \epsilon_{\mu\bar{\mu}} D_{\nu}^{\bar{\mu}}$. Moreover, we get a product of two gamma functions which can be manipulated to give the ratio of two gamma functions which forms part of the closed pomeron propagator $\Pi^c(\alpha's)$ of [21]. The only difference is that we get an extra trigonometric phase factor of the kinematic invariant. This of course can be attributed to the KLT relations which generically require such factors when gluing open string amplitudes to construct closed string ones [23]. The final result is rather long and not very illuminating, therefore we present only the leading behavior

$$\mathcal{V}_{susy}^{GG} = -\text{tr}(\hat{\epsilon}_1 \cdot \hat{\epsilon}_2) e^{-\varphi-\bar{\varphi}} e^{ip \cdot X} \Pi^c(\alpha' p_1 \cdot p_2) (-Q \cdot \partial X \ Q \cdot \bar{\partial} X)^{1-\frac{\alpha'}{2} p_1 \cdot p_2} . \quad (3.52)$$

Finally the pomeron corresponding to the tadpole diagram is

$$\begin{aligned} \mathcal{V}_{susy}^G \sim \hat{\epsilon}_{\mu\bar{\nu}} e^{-\varphi-\bar{\varphi}} e^{2iV \cdot p \cdot X} \Gamma\left(-1 + \frac{\alpha'}{2} p \cdot D \cdot p\right) (-iQ \cdot \partial X)^{-\frac{\alpha'}{2} p \cdot D \cdot p} \\ \left(D^{\mu\bar{\nu}} (-iQ \cdot \partial X) - \psi^\mu (D \cdot \psi)^{\bar{\nu}} \left(-1 + \frac{\alpha'}{2} p \cdot D \cdot p\right) + \dots \right) , \quad (3.53) \end{aligned}$$

where $2Q = \hat{p} - D \cdot \hat{p} = 2N \cdot \hat{p}$.

The results above for the superstring have tachyonic poles, just like the bosonic ones, as commented in [18], which should be cancelled by the OPE with vertices in the rest of the amplitude. These poles would not have been present had we computed the pomeron operators in the 0 picture. Notice that the tadpole operators above have a prefactor typical of an open string pomeron as expected. Remember though that they correspond to the OPE of a closed string operator with itself due to the non-trivial boundary conditions.

4. BCFW shifts in the presence of defects

4.1 A short review of BCFW

The key point of BCFW [5] is that tree level amplitudes constructed using Feynman rules are rational functions of external momenta. Analytic continuation of these momenta on the complex domain turns the amplitudes into meromorphic functions which can be constructed solely by their residues. Since the latter are products of lower point on-shell amplitudes the final outcome is a set of powerful recursive relations.

The simplest complex deformation involves only two external particles whose momenta are shifted as

$$\hat{p}_1(z) = p_1 + qz , \quad \hat{p}_2(z) = p_2 - qz . \quad (4.1)$$

Here $z \in \mathbb{C}$ and to keep the on-shell condition we need $q \cdot p_1 = q \cdot p_2 = 0$ and $q^2 = 0$. In Minkowski spacetime this is only possible for complex q . In particular, if the dimensionality of spacetime is $d \geq 4$, we can choose a reference frame where the two external momenta p_1 and p_2 are back to back with equal energy scaled to 1 [14]

$$p_1 = (1, 1, 0, 0, \dots, 0), \quad p_2 = (1, -1, 0, 0, \dots, 0), \quad q = (0, 0, 1, i, 0, \dots, 0) . \quad (4.2)$$

For gauge bosons the polarizations under a suitable gauge can be chosen as

$$\epsilon_1^- = \epsilon_2^+ = q, \quad \epsilon_1^+ = \epsilon_2^- = q^*, \quad \epsilon_T = (0, 0, 0, 0, \dots, 0, 1, 0, \dots, 0) . \quad (4.3)$$

Under the deformation (4.1) the polarizations become

$$\hat{\epsilon}_1^-(z) = \hat{\epsilon}_2^+(z) = q, \quad \hat{\epsilon}_1^+(z) = q^* + zp_2, \quad \hat{\epsilon}_2^-(z) = q^* - zp_1, \quad \hat{\epsilon}_T(z) = \epsilon_T \quad (4.4)$$

so that they are orthogonal to the shifted momenta.

A general amplitude, which after the deformation becomes a meromorphic function $\mathcal{A}_n(z)$, will have simple poles for those values of z where the propagators of intermediate states go on shell (on the complex plane)

$$\frac{1}{P_J(z)^2} = \frac{1}{P_J(0)^2 - 2zq \cdot P_J} . \quad (4.5)$$

The undeformed amplitude can be computed using Cauchy's theorem:

$$\mathcal{A}_n(0) = \oint_{z=0} \frac{\mathcal{A}_n(z)}{z} dz = - \left\{ \sum \text{Res}_{z=\text{finite}} + \text{Res}_{z=\infty} \right\} . \quad (4.6)$$

As already stated the residues at finite locations on the complex plane are necessarily, due to unitarity, products of lower point tree level amplitudes alas computed at complex on-shell momenta. The residue at infinity can have a similar interpretation in some special cases [12, 13] but in general cannot be written as product of lower point amplitudes. In most cases involving gauge bosons and/or gravitons there is an appropriate choice of the deformed external polarizations such that under a shift of the type (4.1), $\mathcal{A}_n(z)$ vanishes in the limit $z \rightarrow \infty$. In these cases the BCFW relation takes the simple form

$$\mathcal{A}_n(1, 2, 3, \dots, n) = \sum_{r, h(r)} \sum_{k=2}^{n-2} \frac{\mathcal{A}_{k+1}(1, 2, \dots, \hat{i}, \dots, k, \hat{P}_r) \mathcal{A}_{n-k+1}(\hat{P}_r, k+1, \dots, \hat{j}, \dots, n)}{(p_1 + p_2 + \dots + p_k)^2 + m_r^2} , \quad (4.7)$$

where hat is used for the variables which are computed at the residue of the corresponding pole (4.5) and satisfy the physical state condition. Since we have made a complex deformation these are complex momenta unlike the momenta of the rest of the external particles. The summation is over all particle states in the spectrum of the given spectrum and their polarization states.

As was shown in [18, 19], tree level string amplitudes for appropriate regimes of the Mandelstam variables can be made to vanish when $z \rightarrow \infty$. Therefore one expects that these amplitudes satisfy also recursive relations similar to the field theoretic ones (4.7). After this very short review of BCFW we will proceed by considering BCFW shifts in the existence of spacetime defects. The discussion that follows is general and does not confine solely to strings. It could also possibly be useful for cases of macroscopic defects like p-branes or those appearing in brane-world models.

4.2 BCFW shifts in the presence of defects

In the presence of a defect (brane) the fact that momentum is not conserved in the directions normal to the brane¹² implies that there are several novelties when considering BCFW shifts. Let us consider the scattering of a number m of bulk states carrying momenta p_J , $J = 1, \dots, m$ off a brane. In string theory, for instance, this is the case where a number of closed string states are scattered on a D-brane. The generalization where both bulk and brane states (closed and open strings respectively in the D-brane case) are scattered is straightforward. We define the kinematic variables

$$s_{ij} = 2p_i \cdot V \cdot p_j, \quad t_{ij} = -2p_i \cdot p_j \quad (4.8)$$

which can be used to parametrize a general scattering process involving branes. Recall that V is the matrix that projects momenta along the world-volume directions of the brane. Due to momentum conservation along the brane, which reads

$$\sum_{j=1}^m V \cdot p_j = 0, \quad (4.9)$$

the aforementioned invariants are not all independent. Moreover, if the momentum of a particle, say p_k , is along the brane obviously $t_{ik} = -s_{ik}$. As in standard BCFW there is a plethora of multi-particle shifts [6] but we will be interested only in the cases where either one or two momenta are shifted.

One momentum shift. The ability to shift only one momentum without world-volume violating momentum conservation is a novel feature allowed by the defect. For instance, we can shift the momentum of particle 1 as

$$\hat{p}_1(z) = p_1 + zq \quad (4.10)$$

with the on-shell condition and momentum conservation dictating

$$p_1 \cdot q = 0, \quad q^2 = 0, \quad V \cdot q = 0. \quad (4.11)$$

Notice that this implies that q is complex. These shifts do not alter the kinematic invariants s_{ij} and t_{11} and yield a linear z dependence in t_{1i} , $i = 2, \dots, m$. This is actually the minimal scenario where total (i.e. bulk) momentum is not conserved. We will call general such shifts momentum non-conserving.

¹²We mean hard (i.e. shifted) momentum conservation. Soft momenta can be non-conserved without affecting the analysis.

Two momenta shift. Going over to the case where two momenta are affected, we can consider a shift of the type (4.1) but with different deformation vectors q_1, q_2

$$\hat{p}_1(z) = p_1 + q_1 z, \quad \hat{p}_2(z) = p_2 + q_2 z. \quad (4.12)$$

In this case we need to impose the constraints

$$p_1 \cdot q_1 = p_2 \cdot q_2 = 0, \quad q_1^2 = q_2^2 = 0 \quad (4.13)$$

as well as

$$V \cdot q_1 + V \cdot q_2 = 0. \quad (4.14)$$

If $q_1 + q_2 \neq 0$ we have an instance of momentum non-conserving shifts. Since the minimal such scenario is accommodated by a single shift, we will simplify the situation and restrict the two particle shifts to the momentum conserving case $q_1 = -q_2 = q$. Notice that this is the minimal scenario for momentum conserving shifts¹³ and it leaves unchanged the kinematic variables $t_{ij}, i, j = 1, 2$. The variables s_{ik}, t_{ik} with $i = 1, 2$ and $k = 3, \dots, m$ are shifted linearly in z .

We still have the choice of imposing either $V \cdot q = 0$ or $N \cdot q = 0$. If we do impose either one of these conditions then the variables $s_{ij}, i, j = 1, 2$ remain unshifted. We will call such shifts *standard shifts* since they resemble the usual field-theoretic BCFW shifts. Otherwise the s_{ij} for $i, j = 1, 2$ are shifted quadratically in z , which is not desirable if we wish the BCFW deformed amplitude to have simple poles. Higher order poles lead to residues which are non-rational functions of the kinematic variables i.e. have branch-cuts. Since this is not expected for tree level amplitudes they should cancel among the various residues. However, this can make the BCFW procedure rather cumbersome to implement. To avoid this complication we can impose $(V \cdot q)^2 = 0$ (or, equivalently, $(N \cdot q)^2 = 0$) so that the $s_{ij}, i, j = 1, 2$ invariants are shifted only linearly in z . Such shifts we will call *s-shifts*.

Although in the previous discussion we considered scattering only of bulk fields (closed strings), we can obviously make similar two particle shifts for world-volume fields (open strings). For the latter we have the additional constraint that momenta are restricted on the brane and the polarizations can be either parallel or transverse to the brane. Furthermore, we can envisage more general shifts involving the momenta of one bulk and one world-volume field but we will not delve into this.

In the presence of a p -brane the spacetime Lorentz symmetry group $SO(1, d-1)$ is broken to $SO(1, d-1) \rightarrow SO(1, p-1) \times SO(d-p-1)$. We can use the surviving

¹³In terms of our pomeron analysis the minimal shifts, momentum conserving or not, correspond to just two operators approaching each other which, in the symmetric picture of [21], means that the scattering states split into two sets, one with only two operators and another one with the rest. For closed strings on the sphere this holds independently for the holomorphic and anti-holomorphic sector, while on the disc the holomorphic and anti-holomorphic pieces are to be considered as two distinct operators on the disc.

symmetry to bring the two particles to be deformed to the following frame

$$p_1 = (1, \lambda_1, 0, \dots; \lambda_{p+1}, \dots, \lambda_{d-1}) , \quad p_2 = (1, \mu_1, 0, \dots; \mu_{p+1}, \dots, \mu_{d-1}). \quad (4.15)$$

which in the massless case satisfy $-1 + \sum_i \lambda_i^2 = -1 + \sum_i \mu_i^2 = 0$. We have separated tangent and normal directions by a semicolon. The deformation vector q takes the general form

$$q = (n_0, n_1, \dots, n_p; x_{p+1}, \dots, x_{d-1}). \quad (4.16)$$

We see that the shift vector can have components along the world-volume momenta unlike the usual BCFW where a Lorentz transformation can bring them in the form (4.2). This is not possible here due to the presence of the defect which breaks the bulk Lorentz symmetry. Now we discuss in more detail the various possibilities.

4.2.1 One particle shift

In this case we need to find a solution of the conditions (4.11). For a particle with momentum p_1 as in (4.15) we can make a shift, assuming without loss of generality that $x_{p+3} = \dots = x_d = 0$,

$$q = (0, 0, \dots; i \frac{\lambda_{p+2} x_{p+3}}{\sqrt{\lambda_{p+1}^2 + \lambda_{p+2}^2}}, -i \frac{\lambda_{p+1} x_{p+3}}{\sqrt{\lambda_{p+1}^2 + \lambda_{p+2}^2}}, x_{p+3}, 0, \dots) . \quad (4.17)$$

For the polarization vectors

$$\hat{\epsilon}_1^- = q , \quad \hat{\epsilon}_1^+ = q^* + z D \cdot p_1 \quad (4.18)$$

the transversality constraint $\hat{p}_1 \cdot \hat{\epsilon}_1^+ = 0$ demands

$$q \cdot q^* = 2x_{p+3}^2 = -2p_1 \cdot V \cdot p_1 . \quad (4.19)$$

4.2.2 Two particle shifts

As we discussed we distinguish two types of shifts where the momenta of two particles are deformed.

Standard shift. This shift is identical to the shift in (4.1) and changes only s_{ik}, t_{ik} for $i = 1, 2$ and $k = 3, \dots, m$. We can choose either $x_{p+1} = \dots = x_d = 0$ or $n_0 = n_1 = \dots = n_p = 0$. In the first case to satisfy the additional constraints $p_1 \cdot q = p_2 \cdot q = 0$ and $q^2 = 0$ we can select $n_0 = n_1 = n_4 = \dots = n_p = 0$ and $n_2 = i n_3$. We can find similar solutions, albeit more complicated, in the second case provided that $d - p \geq 3$.

S-shift. In this case, in addition to $q \cdot p_1 = q \cdot p_2 = 0$ and $q^2 = 0$ we need to impose

$$(V \cdot q)^2 = 0, \quad \text{with} \quad p_i \cdot V \cdot q \neq 0, i = 1, 2 \quad (4.20)$$

so that we obtain linear z dependence in $s_{ij}, i, j = 1, 2$. A solution of the above conditions is

$$q = (n_0, n_1, n_2, 0, \dots; x_{p+1}, ix_{p+1}, 0, \dots) \quad (4.21)$$

with

$$n_0 = \sqrt{n_1^2 + n_2^2} = \frac{n_1(\mu_1(\lambda_{p+1} + i\lambda_{p+2}) - \lambda_1(\mu_{p+1} + i\mu_{p+2}))}{(\lambda_{p+1} - \mu_{p+1}) + i(\lambda_{p+2} - \mu_{p+2})}, \quad x_{p+1} = \frac{n_0 - n_1\lambda_1}{\lambda_{p+1} + i\lambda_{p+2}}. \quad (4.22)$$

Transversality $\hat{p}_1 \cdot \hat{\epsilon}_1^+ = 0$ of the deformed particle's polarization (4.4) implies

$$q \cdot q^* = -|n_0|^2 + |n_1|^2 + |n_2|^2 = -p_1 \cdot p_2 = \frac{t_{12}}{2} \quad (4.23)$$

which allows us to determine n_2 (for example) and leaves only one undetermined parameter.

For the $1 \rightarrow 1$ scattering process of bulk fields off the brane the above setup cannot be used. This is because we cannot bring the two external particles back to back with equal energy as before. In bulk scattering of course such an amplitude would be just the propagator and has no meaning to make a BCFW shift. But in the presence of branes we can indeed have such processes. In this case we can use the symmetries of tangent and normal directions to bring the two momenta in the form

$$p_1 = (1, \lambda_1, 0, \dots; \lambda_{p+1}, \dots, \lambda_{d-1}), \quad p_2 = (-1, -\lambda_1, 0, \dots; \mu_{p+1}, \dots, \mu_{d-1}). \quad (4.24)$$

The two vectors satisfy momentum conservation along the brane but have arbitrary normal components.

It turns out, see also subsection 5.3, that for the level one states $1 \rightarrow 1$ scattering, the s-shift, which shifts linearly the kinematic invariants, does not change the amplitude for all possible polarization setups. So, in this setup it is more convenient to relax the first condition in (4.20). This means that we will have second order poles when we use the BCFW procedure. Nevertheless, as we mentioned before, the branch-cuts in the residues will cancel leaving a rational function of the external momenta.

A solution of the conditions for this case is

$$q = (n_0, n_1, 0, \dots; x_{p+1}, x_{p+2}, 0, \dots) \quad (4.25)$$

with

$$\begin{aligned}
n_0^2 - n_1^2 &= -\sigma^2 \quad \sigma \in \mathbb{R} , \\
n_0 - n_1 \lambda_1 &= \frac{i\sigma(\lambda_{p+1}\mu_{p+2} - \lambda_{p+2}\mu_{p+1})}{\sqrt{(\lambda_{p+1} + \mu_{p+1})^2 + (\lambda_{p+2} + \mu_{p+2})^2}} , \\
x_{p+1} &= \frac{i\sigma(\lambda_{p+2} + \mu_{p+2})}{\sqrt{(\lambda_{p+1} + \mu_{p+1})^2 + (\lambda_{p+2} + \mu_{p+2})^2}} , \\
x_{p+2} &= -\frac{i\sigma(\lambda_{p+1} + \mu_{p+1})}{\sqrt{(\lambda_{p+1} + \mu_{p+1})^2 + (\lambda_{p+2} + \mu_{p+2})^2}} ,
\end{aligned} \tag{4.26}$$

and the additional constraint from transversality

$$q \cdot q^* = -|n_0|^2 + |n_1|^2 + \sigma^2 = -p_1 \cdot p_2 = \frac{t_{12}}{2} . \tag{4.27}$$

This last relation will be needed in the next section when we will compare the large z behavior of string amplitudes with the pomeron results. Note also that in this case $q \cdot D \cdot p_1 = -q \cdot D \cdot p_2$.

5. Pomerons and large z behavior of open and closed (super) string amplitudes

In this section we will employ the pomeron operators of section 3 to deduce the large z behavior of several string amplitudes. Our analysis for closed strings is very much parallel to the open string case through the use of KLT relations [23]. Nevertheless, there are subtleties which appear and we comment upon them. For open strings the situation is identical to the cases studied in [18, 19] and we will only cite their results. We conclude this section with an explicit example where we compute the BCFW behavior of the superstring scattering amplitude on the disc of two level one closed string states [24, 25, 35, 36] for both the two particle and one particle shifts.

5.1 Bosonic string amplitudes

Open strings, two particle shifts. In this case the relevant pomeron operator is (3.43) and we shift the momenta so that $k = \hat{k}_1 + \hat{k}_2 = k_1 + k_2$ and $2Q = \hat{k}_1 - \hat{k}_2 = k_1 - k_2 + 2qz$. The leading z dependence is extracted when we replace $Q \rightarrow qz$, which implies that $|q| \gg |k_1 - k_2|$. Subleading terms should correspond to α' corrections as one can see in the supersymmetric pomeron computation of [19]. When we insert the pomeron operator, the string amplitude takes the form

$$M^{\mu\nu} = z^{-2\alpha' k_1 \cdot k_2} \left[cz(-\eta^{\mu\nu} + 2\alpha' k^\mu k^\nu) + b(v^\mu k^\nu - v^\nu k^\mu) + \mathcal{O}\left(\frac{1}{z}\right) \right] \tag{5.1}$$

where v^μ, b, c are general functions depending only on the unshifted external momenta and polarizations.

$\epsilon_1 \setminus \epsilon_2$	−	+	T
−	z	$\frac{1}{z}$	$\frac{1}{z}$
+	z^3	z	z
T	z	$\frac{1}{z}$	z
T'	z	$\frac{1}{z}$	$\frac{1}{z}$

Table 1: The leading power in z (modulo the overall factor $z^{-\alpha'(k_1+k_2)^2}$) for adjacent shifts of the all gluon (or transverse scalar) amplitude in the *bosonic string* for all possible polarizations. The polarizations T and T' are orthogonal.

To derive the large z behavior of the S-matrix elements we need to contract the expression above with the polarizations tensors (4.4) and utilize the Ward identities

$$\hat{k}_\mu^1 M^{\mu\nu} \hat{\epsilon}_\nu^2 = 0 \implies q_\mu M^{\mu\nu} \hat{\epsilon}_\nu^2 = -\frac{1}{z} k_\mu^1 M^{\mu\nu} \hat{\epsilon}_\nu^2 \quad (5.2)$$

as well as the transversality identities $q \cdot k = q^* \cdot k = 0$. Actually it is the dependence of the antisymmetric part of (5.1) on k^μ which results into different behavior compared to field theory expectations for some polarization configurations. The result appears in table 1 [19] for the behavior of bosonic string amplitudes under BCFW shifts of level one open string states. The polarizations T' are orthogonal to T and can be either along the brane (gauge bosons) or normal to the brane (transverse scalars).

Closed strings, two particle shifts. The operator in (3.44) can be used to determine the large z behavior of string amplitudes on the sphere as in [18, 19] but also, with similar reasoning, on the disc. On the disc, when the pomeron operator (3.44) is present, there are self-contractions as well as contractions with other operators of the string amplitude. One should make the substitutions (2.12) in order to compute the correlators. For any case the contractions of the pomeron with the other operators in the amplitude will lead to terms like $\alpha' Q \cdot p_k \sim z$ and $\alpha' Q \cdot D \cdot p_k \sim z$, where k runs over all the external particle momenta other than $k = 1, 2$. Therefore a term in the pomeron proportional to $Q^x (DQ)^y$ will lead to an amplitude dependence as z^{x+y} . There are self-contractions as well which lead to terms such as $\alpha' Q \cdot D \cdot Q \sim z$ for all shifts except the one we consider for $1 \rightarrow 1$ scattering. Hence, self-contractions in these amplitudes will lead to a z -dependence of the form z^u with $u < x + y$. Finally, for the $1 \rightarrow 1$ case there are only self-contractions and the shift we consider leads to $\alpha' Q \cdot D \cdot Q \sim z^2$. Moreover, in this particular case $Q \cdot D \cdot p = 0$.

Based on the above, the final behavior of the amplitude is that dictated by KLT

$$M^{\mu\bar{\mu};\nu\bar{\nu}} \sim z^{-\alpha' p_1 \cdot p_2} \left[cz^2 P^{\mu\nu} \bar{P}^{\bar{\mu}\bar{\nu}} + bz(P^{\mu\nu} \bar{A}^{\bar{\mu}\bar{\nu}} + A^{\mu\nu} \bar{P}^{\bar{\mu}\bar{\nu}}) \right. \\ \left. + (A^{\mu\nu;\bar{\mu}\bar{\nu}} + P^{\mu\nu} \bar{B}^{\bar{\mu}\bar{\nu}} + \bar{P}^{\bar{\mu}\bar{\nu}} B^{\mu\nu}) + \mathcal{O}\left(\frac{1}{z}\right) \right] \quad (5.3)$$

$\epsilon_1 \backslash \epsilon_2$	--	-+	++	-T	+T	TT
--	z^2	1	$\frac{1}{z^2}$	1	$\frac{1}{z^2}$	$\frac{1}{z^2}$
-+	z^4	z^2	1	z^2	1	1
++	z^6	z^4	z^2	z^4	z^2	z^2
-T	z^2	1	$\frac{1}{z^2}$	z^2 or 1	1 or $\frac{1}{z^2}$	1 or $\frac{1}{z^2}$
+T	z^4	z^2	1	z^4 or z^2	z^2 or 1	z^2 or 1
TT	z^2	1	$\frac{1}{z^2}$	z^2 or 1	1 or $\frac{1}{z^2}$	$z^2, 1, \text{ or } \frac{1}{z^2}$

Table 2: The leading power in z (modulo the overall factor $z^{-\frac{\alpha'}{2}(p_1+p_2)^2}$) for the large z limit of a an amplitude under a graviton s-shift in the *bosonic string* for all possible polarizations.

where $A^{\mu\nu}$ is an antisymmetric matrix as in (5.1). We can write down the form of the $A_{\mu\nu}$ as a sum of terms which originate from cross-contractions A_1 or self-contractions A_2 of the pomeron vertex

$$A_1^{\mu\nu} \sim (v^\mu p^\nu - v^\nu p^\mu), \quad A_2^{\mu\nu} \sim (q^\mu p^\nu - q^\nu p^\mu). \quad (5.4)$$

The matrix $A_2^{\mu\nu}$ comes from self-contraction of $Q\partial X$ with $\bar{\partial}X^\mu$. The term $B^{\mu\nu}$ can be written as the sum of two terms depending on the operator contraction and has no symmetries. The term $A^{\mu\nu;\bar{\mu}\bar{\nu}}$ comes from the contractions of $K^{\mu\nu}\bar{K}^{\bar{\mu}\bar{\nu}}$. It is obviously antisymmetric in the barred and unbarred indices. In this case the self-contractions give

$$A_2^{\mu\nu;\bar{\mu}\bar{\nu}} \sim (D^{\mu\bar{\mu}}p^\nu(D\cdot p)^{\bar{\nu}} + D^{\nu\bar{\nu}}p^\mu(D\cdot p)^{\bar{\mu}}) - (D^{\mu\bar{\nu}}p^\nu(D\cdot p)^{\bar{\mu}} + D^{\nu\bar{\mu}}p^\mu(D\cdot p)^{\bar{\nu}}) + \dots \quad (5.5)$$

which obviously has the same symmetries as implied by KLT ¹⁴.

Closed strings, one particle shift. This case is the single graviton shift and the relevant pomeron operator is (3.48). The large z behavior is given by the expression

$$M_{\mu\bar{\nu}} = z^{-\frac{\alpha'}{2}p\cdot D\cdot p} \left[cz \left(-D_{\mu\bar{\nu}} + 2\alpha'(V\cdot p)_\mu(V\cdot p)_{\bar{\nu}} \right) + A_{\mu\bar{\nu}} + \mathcal{O}\left(\frac{1}{z}\right) \right] \quad (5.6)$$

where $A^{\mu\bar{\nu}} \sim v^\mu(V\cdot p)^{\bar{\nu}} - (V\cdot p)^\mu(D\cdot v)^{\bar{\nu}}$ is a generic matrix¹⁵ whose explicit dependence on $V\cdot p^\mu$ plays some role in the behavior of amplitudes with ϵ^T polarizations.

¹⁴We notice that self-contractions of pomeron operators can lead to terms which contain the matrix $D^{\mu\bar{\nu}}$, therefore breaking the product of the two independent left and right Lorentz spin symmetries to a diagonal subgroup $SO(1,p) \times \widetilde{SO}(1,p) \times SO(d-p-1) \times \widetilde{SO}(d-p-1)$. Nevertheless these terms affect only the subleading in z expression of the amplitude which, in any case, has the aforementioned symmetries broken. Moreover these terms have the same symmetries in $(\mu, \bar{\mu})$ and $(\nu, \bar{\nu})$ as in the case of supergravity in [14] where no D-branes are present.

¹⁵Nevertheless by pulling a $D_{\mu\bar{\nu}}$ matrix out of $M^{\mu\bar{\nu}}$ we can make it antisymmetric.

$\epsilon_{\mu\bar{\nu}}$	+	-	T
-	z	$\frac{1}{z}$	$\frac{1}{z}$
+	z^3	z	z
T	z	$\frac{1}{z}$	z
T'	z	$\frac{1}{z}$	$\frac{1}{z}$

Table 3: The leading power in z (modulo the overall factor $z^{-\frac{\alpha'}{2}p \cdot Dp}$) for the large z limit of a an amplitude under a graviton one-particle shift in the *bosonic string* for all possible polarizations.

At this point we need to be careful with the polarization assignment since the D-brane inverts the helicity of the right-moving part of the operator. From the gluing of open string operators we expect for the q-light cone polarizations

$$\begin{aligned}
(\hat{\epsilon}D)^{++} &= (q^* + zD \cdot p)(q^* - zp) , \\
(\hat{\epsilon}D)^{-+} &= q(q^* - zp) , \\
(\hat{\epsilon}D)^{+-} &= (q^* + zD \cdot p)q , \\
(\hat{\epsilon}D)^{--} &= qq .
\end{aligned} \tag{5.7}$$

which implies

$$\begin{aligned}
\hat{\epsilon}^{++} &= (q^* + zD \cdot p)(q^* + zD \cdot p) , \\
\hat{\epsilon}^{-+} &= q(q^* + zD \cdot p) , \\
\hat{\epsilon}^{+-} &= (q^* + zD \cdot p)q , \\
\hat{\epsilon}^{--} &= qq
\end{aligned} \tag{5.8}$$

where we have used the fact that the one-particle shift is exclusively in the normal directions (see (4.17)) and therefore $Dq = -q$. The expression above agrees with the transversality condition (4.19). Using Ward identities for gravitons analogous to (5.2) we can deduce the large z behavior of table 3.

5.2 Superstring amplitudes

Open strings, two particle shifts. To determine the large z behavior in this case we use the pomeron vertex operator of (3.49). It is straightforward to derive

$$M^{\mu\nu} = z^{-2\alpha'k_1 \cdot k_2} \left[\eta^{\mu\nu} \left(cz + c' + \mathcal{O}\left(\frac{1}{z}\right) \right) + A^{\mu\nu} + \frac{B^{\mu\nu}}{z} + \dots \right] , \tag{5.9}$$

where $A^{\mu\nu}$ a generic antisymmetric matrix and $B^{\mu\nu}$ a general matrix¹⁶. Notice that

¹⁶Notice that our symmetric OPE expansion does not yield in the pomeron vertex (3.49) the term with \ddot{X} or that with $\dot{\phi}$, as compared to (3.5) and (3.6) of [18]. However, we still get a contribution of order 1 multiplying $\eta^{\mu\nu}$, denoted by c' in the previous formula, from $\eta^{\mu\nu}(Q \cdot \partial X)$ of (3.49) since $Q = k_1 - k_2 + 2qz$. Therefore, we arrive at the same result as that obtained with the pomeron vertex of [18] which was computed with a non-symmetric OPE.

$\epsilon_1 \setminus \epsilon_2$	−	+	T
−	$\frac{1}{z}$	$\frac{1}{z}$	$\frac{1}{z}$
+	z^3	$\frac{1}{z}$	z
T	z	$\frac{1}{z}$	z
T'	z	$\frac{1}{z}$	1

Table 4: The leading power in z (modulo the overall factor $z^{-\alpha'(k_1+k_2)^2}$) for the large z limit of the adjacent shift of an all gluon (transverse scalar) amplitude in the *superstring* for all possible polarizations.

it has similar structure to the bosonic case but it differs in two crucial points: a) the leading term is solely proportional to $\eta^{\mu\nu}$ and b) the subleading antisymmetric matrix is generic and not of the special k -dependent form as in equation (5.1). These crucial differences are responsible for the different behavior for some polarization shifts compared to the bosonic case. In table 4 we present the large z behavior for this case.

Closed strings, two particle shifts. For this case we use the pomeron operator (3.52). After contractions with the remaining operators in the path integral or self-contractions in the $1 \rightarrow 1$ case we arrive at the expected result (that is the product $M_{grav} \sim M_{gauge} \times M_{gauge}$, where M_{gauge} is given in (5.9))

$$M^{\mu\bar{\mu};\nu\bar{\nu}} \sim z^{-\alpha' p_1 \cdot p_2} \left[\eta^{\mu\nu} \eta^{\bar{\mu}\bar{\nu}} z^2 \left(c + \mathcal{O}\left(\frac{1}{z}\right) \right) + bz(\eta^{\mu\nu} \bar{A}^{\bar{\mu}\bar{\nu}} + A^{\mu\nu} \eta^{\bar{\mu}\bar{\nu}}) + (A^{\mu\nu;\bar{\mu}\bar{\nu}} + \eta^{\mu\nu} \tilde{B}^{\bar{\mu}\bar{\nu}} + \eta^{\bar{\mu}\bar{\nu}} B^{\mu\nu}) + \frac{C^{\mu\nu;\bar{\mu}\bar{\nu}}}{z} + \dots \right] \quad (5.10)$$

with the matrices $A^{\mu\nu} \sim (v^\mu l^\nu - v^\nu l^\mu)$ being generic antisymmetric matrices. This is in contrast to the bosonic case where they depend on the pomeron momentum k . Moreover cross- or self-contractions do not change the expected symmetries, from KLT relations, of these matrices. They have the general structure

$$A^{\mu\nu;\bar{\mu}\bar{\nu}} \sim A^{\mu\nu} \bar{A}^{\bar{\mu}\bar{\nu}} ,$$

$$C^{\mu\nu;\bar{\mu}\bar{\nu}} \sim \eta^{\mu\nu} (v^{\bar{\mu}} l^{\bar{\nu}} - v^{\bar{\nu}} l^{\bar{\mu}}) + [(\mu, \nu) \longleftrightarrow (\bar{\mu}, \bar{\nu})] .$$

$A^{\mu\nu;\bar{\mu}\bar{\nu}}$ is antisymmetric in both (μ, ν) and $(\bar{\mu}, \bar{\nu})$ while $C^{\mu\nu;\bar{\mu}\bar{\nu}}$ is a sum of antisymmetric terms in each set. Notice also that as we mentioned in the bosonic case the barred matrices are actually given in terms of unbarred matrices i.e $\bar{G}^{\bar{\mu}\bar{\nu}} = D_{\alpha}^{\bar{\mu}} D_{\beta}^{\bar{\nu}} F^{\alpha\beta}$. In any case the general symmetry properties remain the same and an analysis along the lines of [14] leads to table 5.

$\epsilon_1 \backslash \epsilon_2$	--	-+	++	-T	+T	TT
--	$\frac{1}{z^2}$	$\frac{1}{z^2}$	$\frac{1}{z^2}$	$\frac{1}{z^2}$	$\frac{1}{z^2}$	$\frac{1}{z^2}$
-+	z^2	z^2	$\frac{1}{z^2}$	z^2	1	1
++	z^6	z^2	$\frac{1}{z^2}$	z^4	1	z^2
-T	1	1	$\frac{1}{z^2}$	1	1 or $\frac{1}{z}$	1 or $\frac{1}{z}$
+T	z^4	z^2	1	z^4 or z^3	1	z^2 or z
TT	z^2	1	$\frac{1}{z^2}$	z^2 or z	1 or $\frac{1}{z}$	$z^2, z, \text{ or } 1$

Table 5: The leading power in z (modulo the overall factor $z^{-\frac{\alpha'}{2}(p_1+p_2)^2}$) for the large z limit of a an amplitude under a graviton s-shift in the *superstring* for all possible polarizations.

$\epsilon_{\mu\nu}$	+	-	T
-	$\frac{1}{z}$	$\frac{1}{z}$	$\frac{1}{z}$
+	z^3	$\frac{1}{z}$	z
T	z	$\frac{1}{z}$	z
T2	z	$\frac{1}{z}$	1

Table 6: The leading power in z (modulo the overall factor $z^{-\frac{\alpha'}{2}p \cdot Dp}$) for the large z limit of a an amplitude under a graviton t-shift in the *superstring* for all possible polarizations.

Closed strings, one particle shift. We insert (3.53) and after we rename indices to make manifest the left-right spin Lorenz symmetry, the leading z behavior of the amplitude is

$$M_{\mu\bar{\nu}} = z^{-\frac{\alpha'}{2}p \cdot Dp} \left[D_{\mu\bar{\nu}}(cz + \mathcal{O}(1)) + A_{\mu\bar{\nu}} + \mathcal{O}\left(\frac{1}{z}\right) \right] \quad (5.11)$$

where $A^{\mu\bar{\nu}}$ is an antisymmetric matrix in contrast to the similar matrix in the bosonic which has an explicit dependence on the momenta. The large z behavior of amplitudes under this shift is given in table 6.

5.3 An example: graviton scattering off a D-brane

In this subsection we will work explicitly on the two graviton superstring scattering off a D-brane to verify the behavior advocated in the previous sections under the BCFW shifts. We begin by writing down the amplitude [24, 25, 35, 36]

$$\mathcal{A}(p_1, p_2) \sim -i \frac{\kappa T_p}{2} \frac{\Gamma(-\frac{\alpha'}{4}t) \Gamma(\frac{\alpha'}{2}s)}{\Gamma(1 - \frac{\alpha'}{4}t + \frac{\alpha'}{2}s)} \left(s a_1 + \frac{t}{2} a_2 \right) \quad (5.12)$$

where $t = -(p_1 + p_2)^2 = -2p_1 \cdot p_2$ is the momentum transfer to the p -brane and $s = p_1 \cdot D \cdot p_1 = 2p_1 \cdot V \cdot p_1$ is the momentum flowing parallel to the world-volume of

the brane. The kinematic factors above are:

$$a_1 = \text{tr}(\epsilon_1 \cdot D) p_1 \cdot \epsilon_2 \cdot p_1 - p_1 \cdot \epsilon_2 \cdot D \cdot \epsilon_1 \cdot p_2 - p_1 \cdot \epsilon_2 \cdot \epsilon_1^T \cdot D \cdot p_1 \\ - p_1 \cdot \epsilon_2^T \cdot \epsilon_1 \cdot D \cdot p_1 - p_1 \cdot \epsilon_2 \cdot \epsilon_1^T \cdot p_2 + \frac{s}{2} \text{tr}(\epsilon_1 \cdot \epsilon_2^T) + \left\{ 1 \longleftrightarrow 2 \right\}, \quad (5.13)$$

$$a_2 = \text{tr}(\epsilon_1 \cdot D) (p_1 \cdot \epsilon_2 \cdot D \cdot p_2 + p_2 \cdot D \cdot \epsilon_2 \cdot p_1 + p_2 \cdot D \cdot \epsilon_2 \cdot D \cdot p_2) \\ + p_1 \cdot D \cdot \epsilon_1 \cdot D \cdot \epsilon_2 \cdot D \cdot p_2 - p_2 \cdot D \cdot \epsilon_2 \cdot \epsilon_1^T \cdot D \cdot p_1 + \frac{s}{2} \text{tr}(\epsilon_1 \cdot D \cdot \epsilon_2 \cdot D) \\ - \frac{s}{2} \text{tr}(\epsilon_1 \cdot \epsilon_2^T) - \text{tr}(\epsilon_1 \cdot D) \text{tr}(\epsilon_2 \cdot D) \left(\frac{s}{2} - \frac{t}{4} \right) + \left\{ 1 \longleftrightarrow 2 \right\}. \quad (5.14)$$

Our notation is such that, for example, $p_1 \cdot \epsilon_2 \cdot \epsilon_1^T \cdot D \cdot p_1 = p_1^\mu \epsilon_{2\mu\nu} \epsilon_1^{\lambda\nu} D_{\lambda\rho} p_1^\rho$.

5.3.1 Two particle shift: s-shift

The standard BCFW shift is not applicable here since we have only two scattered states. We have to study therefore the s-shift under which we have

$$\hat{s} = s + 4(p_1 \cdot V \cdot q)z + 2(q \cdot V \cdot q)z^2, \quad \hat{t} = t. \quad (5.15)$$

The gamma function prefactors have an expansion for large $a = \frac{2\sqrt{2}p_1 \cdot V \cdot q}{\sqrt{q \cdot V \cdot q}}$ and $\hat{z} = z\sqrt{2q \cdot V \cdot q}$ as

$$\frac{\Gamma\left(-\frac{\alpha'}{4}t\right)\Gamma\left(\frac{\alpha'}{2}\hat{s}\right)}{\Gamma\left(1-\frac{\alpha'}{4}t+\frac{\alpha'}{2}\hat{s}\right)} \sim \Gamma\left(-\frac{\alpha'}{4}t\right) \hat{z}^{\frac{\alpha'}{2}t-2} \left[1 + a \left(\frac{\alpha'}{4}t - 1 \right) \frac{1}{\hat{z}} + \mathcal{O}\left(\frac{1}{\hat{z}^2}\right) \right]. \quad (5.16)$$

Here we need to make a point. The shift is quadratic in z so we might wonder whether we will have branch cuts in the complex momentum plane when we try to derive BCFW relations. Certainly for tree level amplitudes this would be bizarre. The gamma functions has two poles from the two solutions of $\frac{\alpha'}{2}\hat{s} = -n$, $n \in \mathbb{N}$. The poles are given generically by an expression which contains square roots of the kinematic variables and the two solutions differ by the branch of the square root. At least at the level of the gamma functions the only other part, except the denominator, depends on \hat{z} is $\Gamma\left(1-\frac{\alpha'}{4}t+\frac{\alpha'}{2}\hat{s}\right) \rightarrow \Gamma\left(1-\frac{\alpha'}{4}t-n\right)$, that is independent of the residue position \hat{z} itself. Therefore the final result would be the sum of the two solutions which differ by the branch of the square root and should produce a rational function of the external momenta [37]. Definitely this point needs further clarification but in lack of the BCFW recursion relations themselves further understanding is difficult.

We will study now some of the possible polarization assignments.

- $\hat{\epsilon}_1^{--} = qq$ and $\hat{\epsilon}_2^{++} = qq$. It is easy to see that a_1 in (5.12) vanishes and only a_2 contributes. Calling $K^{\mu\bar{\mu};\nu\bar{\nu}}$ the kinematic factor of the amplitude, we have

$$K^{--;++} \sim (q \cdot D \cdot q)^2 \frac{t^2}{4} \quad (5.17)$$

where we have used the relation $q \cdot D \cdot p_1 = -q \cdot D \cdot p_2$. Upon combining with (5.16) we indeed get

$$M^{-+;++} \sim \hat{z}^{-\alpha' p_1 \cdot p_2 - 2} \left((q \cdot D \cdot q)^2 \frac{t^2}{4} + \dots \right) \quad (5.18)$$

as expected from table 5.

- $\hat{\epsilon}_1^{--} = qq$ and $\hat{\epsilon}_2^{--} = (q^* - zp_1)(q^* - zp_1)$. In this case we use $q^*q = -p_1 \cdot p_2$ and
$$q \cdot D \cdot q = q^* \cdot D \cdot q^*, \quad q^* \cdot D \cdot q = qq^* - q \cdot D \cdot q, \quad q^* \cdot D \cdot p = -q \cdot D \cdot p. \quad (5.19)$$

After some algebra we get

$$K^{--;--} \sim \frac{t^2}{4} (q \cdot D \cdot q - s)^2 + \dots \quad (5.20)$$

Combined with (5.16) it gives the expected behavior for the amplitude $M^{--;--} \sim z^6$.

- $\hat{\epsilon}_1^{++} = (q^* + zp_2)(q^* + zp_2)$ and $\hat{\epsilon}_2^{++} = qq$. This is similar to the previous case and leads to the same result.
- $\hat{\epsilon}_1^{++} = (q^* + zp_2)(q^* + zp_2)$ and $\hat{\epsilon}_2^{--} = (q^* - zp_1)(q^* - zp_1)$. After some straightforward calculations we find

$$K^{++;--} \sim \hat{z}^8 (q \cdot D \cdot q)^2 \frac{t^4}{2} \quad (5.21)$$

which agrees with expectations. The remaining cases can be verified in a similar manner as well.

5.3.2 Single particle shift

In this case the shift of the momentum p_1 results in

$$\hat{s} = s, \quad \hat{t} = t - 2(q \cdot p_2)z = t - \hat{z} \quad (5.22)$$

where $\hat{z} = 2q \cdot p_2 z$. The prefactor expansion is

$$\frac{\Gamma\left(-\frac{\alpha'}{4}\hat{t}\right)\Gamma\left(\frac{\alpha'}{2}s\right)}{\Gamma\left(1-\frac{\alpha'}{4}\hat{t}+\frac{\alpha'}{2}s\right)} \sim \Gamma\left(\frac{\alpha'}{2}s\right) \hat{z}^{-\frac{\alpha'}{2}s-1} \left[1 - \frac{\alpha'}{2\hat{z}^2} \left(1 + \frac{\alpha'}{2}s \right) (s-t) + \mathcal{O}\left(\frac{1}{\hat{z}^3}\right) \right]. \quad (5.23)$$

We also need to choose a gauge for $\epsilon_2^{\mu\nu}$. A convenient choice is to choose a reference momentum vector q and impose the conditions

$$q_\mu e_2^{\mu\nu} = 0. \quad (5.24)$$

- $\epsilon_1^{--} = qq$. The result is given by

$$K^{--} \sim s \operatorname{tr}(\epsilon_2 \cdot D)(q \cdot p_2)^2 \quad (5.25)$$

which leads to the expected $\hat{z}^{-\frac{\alpha'}{2}s-1}$ behavior according to table 6.

- $\epsilon_1^{-+} = q(q^* + D \cdot p_1)$. After some relatively lengthy calculation we get

$$K^{-+} \sim s \left[4s(p_1 \cdot \epsilon_2 \cdot p_1) - 4(p_2 \cdot q)q^* \cdot \epsilon_2 \cdot D \cdot p_2 + 2t(p_2 \cdot D + p_1) \cdot \epsilon_2 \cdot D \cdot p_2 + \operatorname{tr}(\epsilon_2 \cdot D)(4(q \cdot p_2)(q^* \cdot p_2) + t(-2s + t)) + \dots \right] \quad (5.26)$$

which gives again $\hat{z}^{-\frac{\alpha'}{2}s-1}$ as required.

- $\epsilon_1^{++} = (q^* + D \cdot p_1)(q^* + D \cdot p_1)$. We find

$$K^{++} \sim z^3 s \operatorname{tr}(\epsilon_2 \cdot D)(q \cdot p_2)^2 \quad (5.27)$$

again in agreement. The remaining polarization choices require a bit more work but are in agreement with the pomeron analysis expectations.

6. Field theory versus pomeron in the eikonal Regge regime

6.1 General remarks

In this section we will try to identify a field theory which leads to amplitudes with the same large z behavior under BCFW deformations as the pomeron analysis in a special kinematic regime. We distinguish our fields in a “soft” classical background, which describes the unshifted particles, and a “hard” one corresponding to the BCFW shifted particles that carry large momentum parametrized by z . In the limit $z \rightarrow \infty$ we can consider the scattering amplitude as a process where the hard particle is shooting through the soft background. The large z behavior of this amplitude can be determined by analyzing the quadratic fluctuations around the soft background in an appropriate two derivative Lagrangian.

Let us consider the scattering of a number of open and closed strings with momenta k_I and p_J respectively. We now assume that $\sqrt{\alpha'}k_I, \sqrt{\alpha'}p_J \sim \mathcal{O}(\epsilon)$ with $\epsilon \ll 1$ for all I, J , while $\sqrt{\alpha'}q \sim \mathcal{O}(\epsilon^{-1})$ so that $q \cdot k_I$ and $q \cdot p_J$ is of order 1. Subsequently, we take z to be large. Then $\alpha' s_{IJ} \ll 1$ while $\alpha' \hat{s}_{IJ} \sim z$ and the pomeron approximation is valid. We will refer to this particular limit of parameter space as the eikonal Regge (ER) regime [18]. Notice that this regime is clearly distinct from the field theory one which corresponds to $\alpha' \rightarrow 0$ and for which all kinematic variables are small compared to the string scale.

An obvious candidate field theory is an appropriate truncated version of the DBI supplemented by the supergravity effective action¹⁷. In appendix C we give a detailed description of this action. We stress the truncated version of the DBI action because, as pointed out in [14], higher dimension operators such as the multipole type couplings present in the DBI action spoil the good behavior of SYM amplitudes under BCFW deformations. We need to think therefore of a two derivative action which will be the appropriate truncation of the DBI action.

Another important point is that when we consider tree level field theory diagrams constructed by D-brane and bulk fields, not all of them correspond to tree level processes in the string theory side. This is exactly what happens if we consider, for instance, the exchange of a graviton between two branes. It corresponds to an annulus amplitude in string theory and therefore, due to world-sheet duality, to an 1-loop open string amplitude. There is no clear separation of tree and loop level processes in string theory as seen from the field theory side. The only diagrams where bulk fields can be used as intermediate states are those between a brane vertex and a bulk one. Hence, we will need to restore Newton's constant κ_N in the supergravity and brane actions in order to keep track of such diagrams.

We also point out that in the following analysis the soft background fields depend on the coupling constants. These fields are solutions of the corresponding equations of motion that non-linearly complete the sum over the plane waves describing the soft external particles [14]. For example, a background such as the metric can be written as $g_{\mu\nu} = \eta_{\mu\nu} + H_{\mu\nu}(p_i, \epsilon_i, \kappa_N)$. Therefore, a soft background field might actually vanish if we take some coupling constants to zero.

6.2 Field theory analysis of open string BCFW deformation and single bulk field shift

As we explained earlier, the goal is to identify a two derivative action which describes hard particles moving through a soft background and leads to the same large z amplitude behavior as the pomeron analysis in the ER regime. We will assume that in the field theory setup one needs to retain only the DBI terms quadratic in the hard fields. This analysis is presented in appendix C. The soft background has an arbitrary number of soft fields which describe the unshifted particles of a given amplitude. So we use as starting point the Lagrangian (C.13) or its equivalent form for gauge fields which we have not written down.

Non-dynamical metric: pure brane field amplitudes. Let us consider first the case of a non-dynamic metric which corresponds to amplitudes with only world-volume fields. The action in (C.13) takes a very simple form with the induced metric

¹⁷Notice that we ignore all the higher derivative corrections to the DBI action since they would definitely spoil BCFW constructibility.

given by

$$\tilde{g}_{\alpha\beta} = \eta_{\alpha\beta} + \partial_\alpha x^i \partial_\beta x_i. \quad (6.1)$$

We see that the leading coupling in (C.13) is at least quartic in the fields having the two hard fields λ^i and at least two soft ones x^i . Since we have already fixed the gauge we cannot impose in addition the q -light cone gauge and therefore we have a vertex at least quadratic in the soft fields which goes as $\mathcal{O}(z^2)$. On the contrary, the pomeron analysis in (5.9) in the ER regime gives a maximum z dependence $\mathcal{O}(z)$.

The problem above persists in the non-abelian case too. It is known [11, 19] that the pomeron analysis for non-adjacent shifts in color ordered amplitudes produces subleading large z behavior compared to the adjacent case we considered in this paper. So, the upper limit of the pomeron analysis is still the $\mathcal{O}(z)$ behavior of (5.9) in the ER regime. On the other hand, the DBI analysis for the non-abelian theory will result in an $\mathcal{O}(z^2)$ behavior since this is due to the kinetic term common to both abelian and non-abelian actions. In any case, the DBI theory includes higher dimension brane-field operators that lead to increasingly divergent vertices for hard fields therefore spoiling BCFW constructibility [11]. Obviously this behavior is not compatible with the pomeron analysis.

Dynamical metric: mixed brane and bulk field amplitudes. Consider now the case where the metric is dynamic. In the previous paragraph we concluded that brane field operators with dimension higher than quartic are problematic due to the induced metric expansion in the action. One might wonder what will happen if we consider amplitudes which involve the two hard scalars/gauge bosons and n soft gravitons. This case could work since we need only the bulk metric without additional soft brane fields.

The analysis proceeds as in the photon-graviton case of [14]. If we can choose the q -light cone gauge the vertices of order $\mathcal{O}(z^2)$ can be eliminated using the gauge symmetry of the bulk graviton background. Otherwise there is a unique set of diagrams where the two hard scalars are connected via a single cubic vertex to the soft graviton background field. The latter is subsequently connected to the soft gravitons through bulk interactions. In this case we obtain an $\mathcal{O}(z^2)$ dependence which again disagrees with the pomeron result (5.9) in the ER regime.

To track the origin of this discrepancy we can push this analysis a bit further. Consider the amplitude of two scalars carrying momenta k_1, k_2 and one graviton with momentum p . This is given by the expression [42]

$$A \sim \frac{\Gamma(-\alpha't)}{\Gamma(1 - \frac{\alpha'}{2}t)} t \left(-4\alpha'(\epsilon_1 \cdot \epsilon_2) (k_1^\mu \epsilon_{\mu\nu} k_2^\nu) + \frac{\alpha'}{2}(\epsilon_1 \cdot p)(\epsilon_2 \cdot p) \text{tr}(\epsilon D) + \dots \right), \quad (6.2)$$

where we have provided only the relevant piece of the corresponding kinematic factor for our discussion. As in the tachyon discussion in subsection 3.1.3 there is no Regge

behavior for this amplitude¹⁸. Nevertheless, if we use the BCFW standard shift of subsection 4.2 for open strings in the ER limit, from the first term of the amplitude above we get an $\mathcal{O}(z^2)$ behavior in agreement with the aforementioned cubic vertex of DBI. We see that the two scalar/one graviton vertex, which would lead to the unique set of diagrams we mentioned above, cannot be computed using the pomeron method. This leads in essence to the discrepancy between field theory and pomeron analysis.

Moreover if we use the momentum non-conserving deformation of a single graviton of subsection 4.2, we obtain from the second term in (6.2) an $\mathcal{O}(z^2)$ behavior. This pattern for the single particle shifts can be easily generalized to higher point functions. The amplitude of three scalars and one graviton computed in [26], like the tachyon amplitude in (3.19), has no Regge behavior under the single particle shift. Nevertheless, terms such as $(\epsilon \cdot p)^3 \text{tr}(\epsilon D)$ in the kinematic factor lead to $\mathcal{O}(z^3)$ behavior under these BCFW shifts, suggesting we keep terms with more than two derivatives, schematically of the form $x^3 \partial^3 h$, from Taylor expanding the metric in the DBI action as in (C.18). Proceeding further, the one graviton with n scalars amplitude will have a $\mathcal{O}(z^n)$ behavior under the one particle shift making the divergence at complex infinity worse for higher point amplitudes. Therefore, there are shifts of string amplitudes which do not vanish at complex infinity even away from the ER regime, and therefore spoil constructibility under these BCFW deformations.

The final conclusion of this subsection is that the ER regime conjecture is not valid for mixed open-closed string amplitudes without making further assumptions. It might be however that an extra refinement of the ER limit can lead to agreement between a proper truncation of the DBI theory and the pomeron analysis. For instance, the obvious way to achieve agreement is by considering the limit that eliminates all higher dimension operators on the D-branes as well as their coupling to supergravity modes. As we can easily see from (C.9), for Dp -branes with $p \leq 3$ we can take the limit $\alpha' \rightarrow 0$ and $g_s \rightarrow 0$ in order to keep g_{Dp} fixed but $\kappa_N \rightarrow 0$. This is actually the usual decoupling limit in AdS/CFT correspondence [43] where the system is reduced to two independent subsystems: a) an abelian gauge theory along with some free scalars on the D-brane and b) the bulk gravity theory.

Notice that in this decoupling limit we need to consider the non-abelian theory on a stack of D-branes in order to end up with an interacting theory. For gauge bosons the analysis is identical to that of [14] since the DBI action yields the Yang–Mills Lagrangian to quadratic level. The subleading antisymmetric terms in [14] come from the commutators in the gauge field strength. This agrees with the subleading piece of the pomeron computation (5.9). For the case of transverse scalars similar antisymmetric terms come from the commutators of the non-abelian DBI as in [38].

¹⁸For the tachyon amplitude there is no kinematic factor but only a gamma function prefactor which is not shifted under this BCFW deformation and therefore the amplitude does not have any z dependence.

The Lagrangian can be written schematically

$$L = \text{tr} \left(F_{\alpha\beta} F^{\alpha\beta} + (D_\alpha X^i)^2 + [X^i, X^j]^2 \right), \quad (6.3)$$

where $D_\alpha X^i = \partial_\alpha X^i + [A_\alpha, X^i]$. Based on the remarks above the analysis is very similar to the pure YM case which has appeared in the literature and we will not repeat it here. It leads to table 4 in agreement with the pomeron results.

6.3 Field theory analysis of closed string BCFW deformations.

It will be also instructive to try to understand the problem at hand from the point of view of bulk shifts like those in subsection 3.1.3. As in [14] we define vielbeins e, \tilde{e} , with their associated connections $\omega, \tilde{\omega}$ and we write

$$h_{\mu\nu} = e_\mu^a \tilde{e}_\nu^{\tilde{a}} h_{a\tilde{a}}, \quad \nabla_\kappa h_{\mu\nu} = e_\mu^a \tilde{e}_\nu^{\tilde{a}} D_\kappa h_{a\tilde{a}} \quad (6.4)$$

with

$$D_\kappa h_{a\tilde{a}} = \partial_\kappa h_{a\tilde{a}} + \omega_{\kappa a}^b h_{b\tilde{a}} + \tilde{\omega}_{\kappa \tilde{a}}^{\tilde{b}} h_{a\tilde{b}}. \quad (6.5)$$

The action becomes

$$S = \int d^d x \sqrt{-g} \left(G_E^{a\tilde{b}} h_{a\tilde{b}} + \frac{1}{4} g^{\mu\nu} \eta^{ab} \tilde{\eta}^{\tilde{a}\tilde{b}} D_\mu h_{a\tilde{a}} D_\nu h_{b\tilde{b}} - \frac{1}{2} h_{a\tilde{a}} h_{b\tilde{b}} R^{ab\tilde{a}\tilde{b}} \right) + \frac{1}{(\alpha')^2 g_{Dp}^2} \int d^{p+1} \sigma \left(\kappa_N \tilde{h}_{\alpha\beta} T_{DBI}^{\alpha\beta} + \kappa_N^2 \tilde{h}_{\alpha\beta} \tilde{h}_{\gamma\delta} Z_{DBI}^{\alpha\beta;\gamma\delta} \right), \quad (6.6)$$

where $G_E^{a\tilde{b}}$ is the Einstein tensor corresponding to the equations of motion of the background. In the notation above we are trying to make evident the two distinct Lorentz spin symmetries acting on the left and right index of $h_{\mu\nu}$. For the pull-back tensors we use the pull-back vielbeins $\tilde{e}_\alpha^a = \partial_\alpha x^\mu e_\mu^a$. It should be clear from our earlier discussion that the supergravity action, while it naively looks independent of κ_N , it does have an implicit dependence through the graviton background fields.

For the momentum conserving two particle shifts the field theory analysis requires tadpole cancellation. The term linear in h_{ab} is a tadpole due to the presence of the boundary state meaning that the DBI action sources the Einstein equations

$$G_E^{a\tilde{b}} = T_p \delta^{d-p-1} (x^i - x_0^i) \tilde{T}_{DBI}^{a\tilde{b}}. \quad (6.7)$$

We assume that the deformation vector q_μ is along the $+$ direction and we choose the light cone gauge

$$\omega_{ab}^+ = \tilde{\omega}_{\tilde{a}\tilde{b}}^+ = g^{++} = g^{+\kappa} = 0 \quad \text{and} \quad g^{+-} = 1, \quad (6.8)$$

so that there are no $\mathcal{O}(z^2)$ vertices.

For the moment let us ignore the quadratic graviton contribution of the DBI action. Then the analysis proceeds exactly as in [14]. The order $\mathcal{O}(z)$ vertices preserve both the spin Lorentz symmetries – except for the unique set of bulk diagrams

(not to be confused with the unique brane diagrams which play role for the DBI couplings) that give contributions up to $\mathcal{O}(z^2)$. These $\mathcal{O}(z^2)$ terms come from the two derivative part of the Lagrangian, do not break either of the left or right spin “Lorentz” invariance, and thus are proportional to $\eta^{ab}\tilde{\eta}^{\tilde{a}\tilde{b}}$. The order $\mathcal{O}(z)$ terms that violate the symmetry and come from a derivative on h and a single ω or $\tilde{\omega}$ insertion have the form $\eta^{ab}\tilde{A}^{\tilde{a}\tilde{b}} + A^{ab}\tilde{\eta}^{\tilde{a}\tilde{b}}$ where A and \tilde{A} are antisymmetric. The $\mathcal{O}(1)$ parts of the amplitude will receive contributions from both the bulk and the DBI action. Most of the pieces and their origin is the same as in [14] and we will refer the reader to this paper for further details.

The bulk Riemann tensor term of the action is sourced by the stress energy tensor (6.7) of the DBI action. The Einstein equations give a backreacted metric and a corresponding Riemann tensor. This will not affect the large z behavior of the amplitudes since they depend on the symmetries of the Riemann tensor rather than its actual form. Nevertheless the $D^{a\tilde{a}}$ dependence of the matrix elements of $M^{a\tilde{a}b\tilde{b}}$ in the pomeron analysis (5.10) has its origin on the brane contribution to the background metric. The brane with all its fields is part of the soft background through which the hard gravitons propagate¹⁹. Thus we find

$$M^{a\tilde{a}b\tilde{b}} = cz^2\eta^{ab}\tilde{\eta}^{\tilde{a}\tilde{b}} + z\left(\eta^{ab}\tilde{A}^{\tilde{a}\tilde{b}} + A^{ab}\tilde{\eta}^{\tilde{a}\tilde{b}}\right) + A^{ab\tilde{a}\tilde{b}} + \eta^{ab}\tilde{B}^{\tilde{a}\tilde{b}} + B^{ab}\tilde{\eta}^{\tilde{a}\tilde{b}} + \frac{1}{z}C^{ab\tilde{a}\tilde{b}} + \dots \quad (6.9)$$

where A^{ab} is an antisymmetric matrix, B^{ab} is an arbitrary matrix, and $A^{ab\tilde{a}\tilde{b}}$ is antisymmetric in (ab) and $(\tilde{a}\tilde{b})$. This agrees with the pomeron result (5.10). From the point of view of field theory it is quite remarkable that this symmetry structure is precisely what we get by squaring the Yang-Mills result (5.9). Instead, this phenomenon is perfectly understood in string theory in light of the KLT relations [23].

Finally, notice that if we do not ignore the quadratic graviton contribution of the DBI action we face an obvious problem. The tensor $Z_{DBI}^{\alpha\beta;\gamma\delta}$ has no symmetry in its world-volume indices. This term gives a contribution to the $\mathcal{O}(1)$ terms of (6.9) but it does not have the same symmetries as any of them and so it will spoil agreement with the pomeron result (5.10). Therefore, we need to consider the decoupling limit once more. For $p < 3$ this limit requires $g_s \rightarrow 0$ as well, while for $p = 3$ we can keep g_s fixed while sending $\kappa_N \rightarrow 0$ and keeping g_{D3} fixed. Then one notices that the tadpole term in (C.12) survives and the quadratic one vanishes in accord with (5.10).

¹⁹Considering only bosonic fields for simplicity we can see that the D-brane contributions to (5.10) are of subleading order, $\mathcal{O}(z)$ and below, and have the same structure with the $\mathcal{O}(z)$ terms of the pure gravity analysis. This is very similar to equation (4.16) of [15] which has given the large z analysis of gravity coupled to matter.

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Appendix A: Details on pomeron derivations

In this appendix we give a detailed derivation of the bosonic closed string pomeron vertex for level 1 states. The supersymmetric case follows in a similar manner. In the formulas below we show only the leading terms in the shifted momenta. Subleading terms in \hat{p}_i or \hat{k}_i are represented by ellipsis. We begin with the OPE

$$\begin{aligned} \partial X^\mu \bar{\partial} X^\nu e^{i\hat{p}_1 \cdot X} \left(\frac{w}{2} \right) \partial X^\rho \bar{\partial} X^\sigma e^{i\hat{p}_2 \cdot X} \left(-\frac{w}{2} \right) &\sim e^{ip \cdot X + iwQ \cdot \partial X + i\bar{w}Q \cdot \bar{\partial} X}(0) |w|^{\alpha' p_1 \cdot p_2} \\ &\left(\frac{-\eta^{\mu\rho} + \frac{\alpha'}{2} \hat{p}_2^\mu \hat{p}_1^\rho}{w^2} + i \frac{\partial X^\mu \hat{p}_1^\rho - \partial X^\rho \hat{p}_2^\mu}{w} + \dots \right) \times \left(\frac{-\eta^{\nu\sigma} + \frac{\alpha'}{2} \hat{p}_2^\nu \hat{p}_1^\sigma}{\bar{w}^2} + i \frac{\bar{\partial} X^\nu \hat{p}_1^\sigma - \bar{\partial} X^\sigma \hat{p}_2^\nu}{\bar{w}} + \dots \right) \end{aligned} \quad (\text{A.1})$$

where, as in the main text, $p = \hat{p}_1 + \hat{p}_2$ and $2Q = \hat{p}_1 - \hat{p}_2$. Now we make the substitutions

$$W = -wQ \cdot \partial X, \quad \bar{W} = -\bar{w}Q \cdot \bar{\partial} X \quad (\text{A.2})$$

and we integrate over the position $W = re^{i\theta}$ on the sphere using the integral formulas of Bessel functions

$$\int_0^{2\pi} d\theta e^{-2ir \cos \theta} = 2\pi J_0(2r), \quad \int_0^{2\pi} d\theta e^{i\theta} e^{-2ir \cos \theta} = -2\pi i J_1(2r), \quad (\text{A.3})$$

$$\int_0^\infty dr r^a J_0(2r) = \frac{1}{2} \frac{\Gamma\left(\frac{1+a}{2}\right)}{\Gamma\left(\frac{1-a}{2}\right)}, \quad \int_0^\infty dr r^a J_1(2r) = \frac{1}{2} \frac{\Gamma\left(1 + \frac{a}{2}\right)}{\Gamma\left(1 - \frac{a}{2}\right)}. \quad (\text{A.4})$$

Defining $P^{\mu\nu}$ and $K^{\mu\nu}$ as in (3.45) and after some trivial manipulations we arrive at the final formula

$$\begin{aligned} M^{\mu\nu;\rho\sigma} &= \frac{\Gamma\left(\frac{\alpha'}{2} p_1 \cdot p_2 - 1\right)}{\Gamma\left(2 - \frac{\alpha'}{2} p_1 \cdot p_2\right)} e^{ip \cdot X} (Q \cdot \partial X Q \cdot \bar{\partial} X)^{-\frac{\alpha'}{2} p_1 \cdot p_2} \\ &\left[(Q \cdot \partial X) P_{\mu\rho} - K_{\mu\rho} \left(\frac{\alpha'}{2} p_1 \cdot p_2 - 1 \right) \right] \times \left[(Q \cdot \bar{\partial} X) \bar{P}_{\nu\sigma} - \bar{K}_{\nu\sigma} \left(\frac{\alpha'}{2} p_1 \cdot p_2 - 1 \right) \right] \end{aligned} \quad (\text{A.5})$$

We show now how we can derive (3.51). We start with (3.50) which, using the standard technique of rewriting the vertex operators as exponentials in terms of

Grassmann variables, becomes

$$\begin{aligned} & \hat{\epsilon}_{1;\mu} \hat{\epsilon}_{2;\nu} e^{-\varphi(0)} e^{2ik \cdot X(0) + 2iy(\hat{k}_1 - \hat{k}_2) \cdot \partial X(0)} (2y)^{2\alpha' k_1 \cdot k_2} (1 + y\dot{\phi}(0)) \\ & \left[-i \frac{\alpha'}{2} \left(\frac{\hat{k}_2^\mu \psi^\nu(0) - \hat{k}_1^\nu \psi^\mu(0)}{y} \right) + i \frac{\alpha'}{2} (\hat{k}_2^\mu \dot{\psi}^\nu(0) + \hat{k}_1^\nu \dot{\psi}^\mu(0)) + \left(\partial X^\mu + i\alpha' (\hat{k}_1 \cdot \psi) \psi^\mu \right) \psi^\nu(0) \right. \\ & \quad \left. - i \frac{\alpha'}{2} \frac{\eta^{\mu\nu}}{y} \left((\hat{k}_1 \cdot \psi) + y(\hat{k}_1 \cdot \dot{\psi}) \right) (0) \right] . \end{aligned} \quad (\text{A.6})$$

We make now the substitution $Y = yQ \cdot \partial X$ and the integrations are usual gamma function integrals. In addition, we use the physical state conditions $\hat{\epsilon}_i^\mu \hat{k}_{i;\mu} = 0$ to rewrite

$$\hat{\epsilon}_1^\mu \hat{k}_{2;\mu} = \hat{\epsilon}_1^\mu k_\mu , \quad \hat{\epsilon}_2^\mu \hat{k}_{1;\mu} = \hat{\epsilon}_2^\mu k_\mu . \quad (\text{A.7})$$

The final result after the integration over y is

$$\begin{aligned} & \hat{\epsilon}_{1;\mu} \hat{\epsilon}_{2;\nu} e^{-\varphi(0)} e^{2ik \cdot X(0)} \Gamma(-1 + 2\alpha' k_1 \cdot k_2) (Q \cdot \partial X)^{-1-2\alpha' k_1 \cdot k_2} \times \\ & \times \left(\eta^{\mu\nu} (Q \cdot \partial X) (\hat{k}_1 \cdot \psi) + A_1^{\mu\nu} + \eta^{\mu\nu} C + (Q \cdot \partial X) A_2^{\mu\nu} + \dots \right) \end{aligned} \quad (\text{A.8})$$

where

$$\begin{aligned} A_1^{\mu\nu} &= -2(\hat{k}_1 \cdot \psi) \psi^\mu \psi^\nu (-1 + 2\alpha' k_1 \cdot k_2) , \quad A_2^{\mu\nu} = -(\psi^\mu k^\nu - \psi^\nu k^\mu) , \\ C &= (-1 + 2\alpha' k_1 \cdot k_2) \left((\hat{k}_1 \cdot \psi) \dot{\phi} + (\hat{k}_1 \cdot \dot{\psi}) \right) . \end{aligned} \quad (\text{A.9})$$

Appendix B: Open-closed mixing pomeron

Assume that the closed string tachyon has only world-volume momenta, $\alpha' \hat{p} \cdot D \cdot \hat{p} = \alpha' \hat{p} \cdot V \cdot \hat{p} = 4$. The relevant OPE takes the form

$$\begin{aligned} & : e^{i\hat{p} \cdot X(w)} : : e^{iD\hat{p} \cdot X(\bar{w})} : : e^{2i\hat{k} \cdot X(z)} : \sim \\ & (w - \bar{w})^{\frac{\alpha'}{2} \hat{p} \cdot D \cdot \hat{p}} (w - z)^{\alpha' \hat{p} \cdot \hat{k}} (\bar{w} - z)^{\alpha' \hat{p} \cdot D \cdot \hat{k}} : e^{i\hat{p} \cdot X(w) + iD\hat{p} \cdot X(\bar{w}) + 2i\hat{k} \cdot X(z)} : . \end{aligned} \quad (\text{B.1})$$

Using $D\hat{p} = V\hat{p} = \hat{p}$ and Cartesian coordinates for the upper half-plane $w = x + iy$ as well as expanding $X(z)$ around x and setting $z = x + R$ yields the pomeron operator

$$\mathcal{V}^{T_c T_o} \sim \int_{-\infty}^{+\infty} dR \int_0^\infty dy y^2 (R^2 + y^2)^{\alpha' p \cdot \hat{k}} e^{2iR\hat{k} \cdot \partial X} e^{2i(\hat{p} + \hat{k}) \cdot X(x)} . \quad (\text{B.2})$$

The integration above gives

$$\mathcal{V}^{T_c T_o}(\hat{k}, \hat{k} + V \cdot \hat{p}) \sim (\hat{k} \cdot \partial X)^{-2\alpha' \hat{k} \cdot \hat{p} - 4} \frac{\Gamma\left(-\frac{3}{2} - \alpha' \hat{p} \cdot \hat{k}\right) \Gamma(4 + 2\alpha' \hat{p} \cdot \hat{k})}{\Gamma(-\alpha' \hat{p} \cdot \hat{k}) \Gamma\left(\frac{1}{2} + \alpha' \hat{p} \cdot \hat{k}\right) \Gamma\left(\frac{1}{2} - \alpha' \hat{p} \cdot \hat{k}\right)} e^{2i(V\hat{p} + \hat{k}) \cdot X(x)} . \quad (\text{B.3})$$

Finally, we can insert this operator on the disc along with two open string tachyons to calculate

$$\left\langle \mathcal{V}^{T_c T_o}(\hat{k}_3, \hat{k}_3 + V \cdot \hat{p}) V^{T_o}(k_1) V^{T_o}(k_2) \right\rangle \quad (\text{B.4})$$

and we get the result (3.25) for $\alpha'(V \cdot \hat{p})^2 = 4$, $2\alpha' \hat{p} \cdot \hat{k} = -s - 3$.

Appendix C: The candidate effective actions

In this appendix we present the two low energy effective actions, supergravity and DBI, that can serve as a starting point of our analysis. For the supergravity we consider an expansion $G_{\mu\nu} = g_{\mu\nu} + 2\kappa_N h_{\mu\nu}$ in terms of gravitational fluctuations $h_{\mu\nu}$ about an arbitrary classical background $g_{\mu\nu}$. We consider only the NS-NS sector and moreover we ignore the Kalb–Ramond field for simplicity. We write the action plus de-Donder gauge fixing terms. Furthermore, we make a field redefinition to decouple the dilaton from the physical graviton field, i.e. we go to the Einstein frame²⁰

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + g_{\mu\nu} \sqrt{\frac{2}{D-2}} \phi, \quad \phi \rightarrow \frac{1}{2} g^{\mu\nu} h_{\mu\nu} + \sqrt{\frac{D-2}{2}} \phi \quad (\text{C.1})$$

so that the gravity Lagrangian simply becomes

$$L = \sqrt{-g} \left(\frac{1}{4} g^{\mu\nu} g^{\kappa\rho} g^{\lambda\sigma} \nabla_\mu h_{\kappa\lambda} \nabla_\nu h_{\rho\sigma} - \frac{1}{2} h_{\kappa\lambda} h_{\mu\nu} R^{\lambda\mu\kappa\nu} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right). \quad (\text{C.2})$$

We will consider only amplitudes involving gravitons since this suffices for our purposes. Therefore we will drop the (re-defined) dilaton field, since it decouples from the amplitudes we are interested in.

The abelian DBI action for a Dp -brane in the Einstein frame ($d = 10$) takes the form

$$S_{DBI} = -T_p \int d^{p+1} \sigma \, e^{\frac{p-3}{4} \Phi} \sqrt{-\det(\tilde{G}_{\alpha\beta} + e^{-\Phi/2} \tilde{B}_{\alpha\beta} + 2\pi\alpha' e^{-\Phi/2} F_{\alpha\beta})}. \quad (\text{C.3})$$

The fields \tilde{G}_{ab} and \tilde{B}_{ab} are pull-backs

$$\tilde{G}_{\alpha\beta} = G_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu, \quad \tilde{B}_{\alpha\beta} = B_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu \quad (\text{C.4})$$

of the bulk fields $G_{\mu\nu}, B_{\mu\nu}$. The fields $X^\mu(\sigma)$, $\mu = 0, 1, \dots, d-1$ describe the embedding of the p -dimensional world-volume of the brane, which is parametrized by σ^α , $\alpha = 0, \dots, p$, in the ambient spacetime. The two vector fields $\partial_\alpha X^\mu$ and ξ_i^μ , $i = p+1, \dots, 9$ define tangent and normal bundle frames respectively and satisfy the relations

$$\xi_i^\mu \xi_j^\nu G_{\mu\nu} = \delta_{ij}, \quad \xi_i^\mu \partial_\alpha X^\nu G_{\mu\nu} = 0 \quad (\text{C.5})$$

where δ_{ij} is the normal bundle metric. Using these frames we pull-back tensors from the ambient space to the tangent and normal bundle.

In the non-abelian action the various brane fields become matrix-valued functions and the symmetrized trace prescription is applied on the usual DBI. Moreover,

²⁰We use middle alphabet Greek letters for bulk indices.

there are extra terms involving commutators of the fields. We will not give the form of the non-abelian Lagrangian here since it will not be necessary for what follows.

We choose to work in the static gauge where the world-volume coordinates σ^α coincide with the bulk coordinates X^μ for $\mu = 0, \dots, p$, which implies

$$\partial_\alpha X^\beta = \delta_\alpha^\beta, \quad G_{\mu\nu}(X^\kappa) = G_{\mu\nu}(\sigma, X^i(\sigma)) . \quad (\text{C.6})$$

The fields X^i describe transverse fluctuations of the brane. In this gauge (C.4) takes the form

$$\tilde{G}_{\alpha\beta} = G_{\alpha\beta} + 2G_{i(\alpha}\partial_{\beta)}X^i + G_{ij}\partial_\alpha X^i\partial_\beta X^j \quad (\text{C.7})$$

and similarly for the antisymmetric tensor field.

Now, we expand the DBI action to second order in the hard fields $h_{\mu\nu}$, λ_i and $f_{\alpha\beta}$. Since we will consider for simplicity only graviton amplitudes we can set $B_{\mu\nu} = 0$ and $\Phi = 0$. The various fields are expanded around the soft background $g_{\mu\nu}$, x^i and A_α as follows

$$G_{\mu\nu} = g_{\mu\nu} + 2\kappa_N h_{\mu\nu}, \quad X^i = x^i + \frac{1}{\sqrt{T_p}}\lambda^i, \quad \tilde{A}_\alpha = A_\alpha + \frac{1}{\sqrt{T_p}2\pi\alpha'}a_\alpha . \quad (\text{C.8})$$

Notice that $g_{\mu\nu}$ and $h_{\mu\nu}$ depend on X^i and therefore the total fluctuation in $G_{\mu\nu}$ includes also λ .

It will be useful to write down the coupling constants that appear in the action in terms of Newton's constant κ_N , the coupling of the gauge fields on the brane g_{Dp} and α' . Moreover we give the relations among the open g_o and closed g_c string couplings which appear in the definitions of the string vertex operators. We have the following relations [39, 40] which will be useful in our discussion

$$g_c = 2\pi\kappa_N \sim g_s(\alpha')^2, \quad \frac{g_o^2}{g_c} \sim (\alpha')^2, \\ T_p \sim \frac{1}{(\alpha')^2 g_{Dp}^2}, \quad g_{Dp}^2 \sim g_s(\alpha')^{(p-3)/2}, \quad (\text{C.9})$$

where g_s is the asymptotic closed string coupling and we have ignored the exact numerical coefficients.

For simplicity we will restrict ourselves to the case where the gauge field are turned off. In other words we will not consider amplitudes which involve gauge fields either soft or hard²¹. The quadratic action is of the schematic form

$$S_{DBI}^{quad} = S_{DBI}^{gr} + S_{DBI}^{sc} + S_{DBI}^{mix}, \quad (\text{C.10})$$

²¹Actually, if we keep the gauge fields the formulas below are written in terms of the open string metric [41] instead of the induced one.

where the various contributions take their name from the hard fields they involve. From the expansion of the square root

$$\sqrt{\det(\tilde{g}_{\alpha\beta} + \tilde{h}_{\alpha\beta})} = \sqrt{\det \tilde{g}_{\alpha\beta}} \left(1 + \frac{1}{2} \tilde{h}_{\alpha}^{\alpha} - \frac{1}{4} \tilde{h}_{\alpha\beta} \tilde{h}^{\beta\alpha} + \frac{1}{8} (\tilde{h}_{\alpha}^{\alpha})^2 + \dots \right) \quad (\text{C.11})$$

we obtain the two graviton action

$$S_{DBI}^{gr} \sim \frac{1}{(\alpha')^2 g_{Dp}^2} \int d^{p+1} \sigma \left(\kappa_N \tilde{h}_{\alpha\beta} T_{DBI}^{\alpha\beta} + \kappa_N^2 \tilde{h}_{\alpha\beta} \tilde{h}_{\gamma\delta} Z_{DBI}^{\alpha\beta;\gamma\delta} \right), \quad (\text{C.12})$$

where of course all indices are raised and lowered with the $\tilde{g}_{\alpha\beta}$ metric²². The linear coupling is proportional to the brane stress energy tensor $T_{DBI}^{\alpha\beta}$. The quadratic term has a coefficient $Z_{DBI}^{\alpha\beta;\gamma\delta}(\tilde{g})$ with no symmetry in its indices.

The two scalar action takes the form

$$S_{DBI}^{sc} \sim \int d^{p+1} \sigma \sqrt{\det \tilde{g}} \tilde{g}^{\alpha\beta} g^{ij} \partial_{\alpha} \lambda_i \partial_{\beta} \lambda_j. \quad (\text{C.13})$$

Then, using vielbeins to expand the metric in the normal and tangent bundle

$$g_{\mu\nu} = e_{\mu}^{\tilde{i}} e_{\nu}^{\tilde{j}} \delta_{\tilde{i}\tilde{j}} + e_{\mu}^{\alpha} e_{\nu}^{\beta} \eta_{\alpha\beta}, \quad e_{\mu}^{\tilde{i}} e^{\alpha\mu} = 0 \quad (\text{C.14})$$

and making the field redefinition $\lambda_i = \tilde{e}_i^{\tilde{\lambda}} \lambda_{\tilde{\lambda}}$ we arrive at the simple geometrical action

$$S_{DBI}^{sc} \sim \int d^{p+1} \sigma \sqrt{\det \tilde{g}} \tilde{g}^{\alpha\beta} D_{\alpha} \lambda^{\tilde{i}} D_{\beta} \lambda_{\tilde{i}}. \quad (\text{C.15})$$

The covariant derivative above is defined using the normal bundle connection $\omega_{N;\alpha}^{[\tilde{i}\tilde{j}]}$ (see for instance [26])

$$D_{\alpha} \lambda_{\tilde{i}} = \partial_{\alpha} \lambda_{\tilde{i}} + (\omega_{N;\alpha}^{[\tilde{i}\tilde{j}]} - \partial_{\alpha} x^{\mu} \Gamma_{\mu\kappa}^{\rho} e_{\tilde{i}}^{\kappa} e_{\rho}^{\tilde{j}}) \lambda_{\tilde{j}} \quad (\text{C.16})$$

as it is the natural object which commutes with the vielbeins $e_{\tilde{i}}^{\tilde{\lambda}}$. The action mixing bulk and brane modes takes the form

$$S_{DBI}^{mix} \sim \frac{1}{\alpha' g_{Dp}} \kappa_N \int d^{p+1} \sigma \sqrt{\det \tilde{g}} \tilde{g}^{\alpha\beta} g^{\mu\nu} (\partial_{\alpha} \lambda_{\mu} \partial_{\beta} x^{\kappa} h_{\kappa\nu} + \lambda_{\mu} \partial_{\alpha} x^{\kappa} \partial_{\beta} x^{\lambda} \partial_{\nu} h_{\kappa\lambda}). \quad (\text{C.17})$$

This can also be written in an equivalent form using the normal bundle vielbeins. The second term in the action above originates from Taylor expanding the metric in (C.6) around the background in (C.8)

$$G_{\mu\nu}(\sigma, X^i) = G_{\mu\nu}(\sigma, x^i) + \lambda^i \partial_i G_{\mu\nu}(\sigma, x^i) + \dots \quad (\text{C.18})$$

The full action needed for our analysis is given by the sum of the actions in (C.2) and (C.10).

²²Notice that $\tilde{g}_{\alpha\beta}$ and $\tilde{h}_{\alpha\beta}$ are the pull-backs of $g_{\mu\nu}$ and $h_{\mu\nu}$ evaluated at $\lambda^i = 0$.

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